Small-signal-equivalent circuits for a semiconductor laser

Osman Kibar, Daniel Van Blerkom, Chi Fan, Philippe J. Marchand, and Sadik C. Esener

Passive electrical circuits whose voltage and current equations are exactly equivalent to the small-signal rate equations of a semiconductor laser are derived to model an electrically modulated laser (verified to be the same as that given in the literature), an optically modulated laser (i.e., a laser used as an optical amplifier), and a multimode laser. These circuits offer a fast and efficient simulation tool with little computational complexity in which the small-signal assumption (i.e., small modulation range) is neither violated nor insufficient for the simulation. © 1998 Optical Society of America

OCIS code: 140.5900.

1. Introduction

Semiconductor lasers have been extensively modeled in terms of equivalent electrical circuits\(^1\)\(^-\)\(^5\) to simulate their large-signal behavior. However, because the laser behavior includes nonlinear effects, accurate models tend to be computationally demanding. For simulations for which speed is favored over accuracy or the operation of the laser is limited to a small modulation range, it is more advantageous to model the small-signal behavior of the laser. In this case the laser equations are reduced to linear differential equations, and the electrical circuits that model the laser are significantly simplified. Previously, based on the impedance characteristics of a laser, an equivalent electrical circuit was proposed to model the small-signal behavior of lasers in the presence of small perturbations.\(^7\) This circuit has been further modified to include the effects of lateral carrier diffusion\(^8\) and of package parasitics\(^9\) under electrical modulation.

In this paper we exploit the similarities between semiconductor lasers and electrical RLC circuits to repeat the derivations of the above-described electrical circuits, which exactly model the small-signal behavior of semiconductor lasers under electrical modulation. We then extend our derivations to include optical modulation (i.e., when a laser is used as an optical amplifier) and multimode lasers in which the secondary modes can be longitudinal, spatial, or polarization modes. In Section 2 we state the small-signal rate equations and outline the procedure to convert these equations into the current and the voltage equations of RLC circuits. This procedure is then applied to electrical modulation, and the resulting circuit is compared with the previously reported circuits for verification of our model. In Sections 3 and 4 we modify the equations and our electrical circuit to include optical modulation as well as multimode operation of a laser. Section 5 reviews the results of the modeling and concludes the paper.

2. Modeling Small-Signal Laser Behavior

Rate equations can be used for a simple treatment of the frequency-domain or time-domain behaviors of the electron and the photon numbers in a laser cavity. The reader is referred to any textbook on lasers for a discussion of the rate equations and their validity for various laser phenomena (see, e.g., Ref. 10). The general rate equations for a single-mode laser are

\[
P(t) = GP(t) - \gamma P(t) + R_{sp}, \tag{1a}
\]

\[
N(t) = \frac{I_{bias}}{q} - \gamma_e N(t) - GP(t), \tag{1b}
\]

where \(P\) and \(N\) are the total number of photons in the lasing mode and electrons in the excited state inside the cavity, respectively. The dots over \(P\) and \(N\) represent time derivatives. The three terms on the right-hand side (RHS) of Eq. (1a) are, in order, the stimulated emission rate, the total photon-loss rate including the loss through the mirrors, and the spontaneous emission rate into the lasing mode. The three terms on the RHS in Eq. (1b) are, in order, the injection rate of electrons into the gain medium, the
electron-loss rate (including nonradiative recombination, spontaneous emission, and Auger recombination), and the loss rate of electrons that results from stimulated emission.

The linearized small-signal rate equations in the presence of a small perturbation (i.e., if we assume $\delta P \ll P$ and $\delta N \ll N$) are\(^{10}\)

$$\delta P(t) = -\Gamma_p \delta P(t) + \sigma_{N\rightarrow p} \delta N(t), \quad (2a)$$
$$\delta N(t) = -\Gamma_N \delta N(t) - \sigma_{p\rightarrow N} \delta P(t), \quad (2b)$$

where $\Gamma_p$ and $\Gamma_N$ are the decay rates of fluctuations of the photon and the electron numbers, respectively, and $\sigma_{N\rightarrow p}$ and $\sigma_{p\rightarrow N}$ are the coupling coefficients between the photon- and the electron-number fluctuations, respectively. Note that the electron and the photon numbers in Eqs. (2) $\delta N$ and $\delta P$ refer to the deviations from the equilibrium values at a given bias of the laser. At a given bias point, which fixes $P$ and $N$, the four coefficients in Eqs. (2) are given as

$$\Gamma_p = \frac{R_{ep}}{P} - G_p P, \quad (3a)$$
$$\Gamma_N = \gamma_e + \gamma_{en} N + G_N P, \quad (3b)$$
$$\sigma_{N\rightarrow p} = G_N P + R_{pN}, \quad (3c)$$
$$\sigma_{p\rightarrow N} = G + G_p P, \quad (3d)$$

where the subscripts $P$ or $N$ on the RHS of the equations mean the derivatives with respect to $P$ or $N$, respectively.

At this point we note that the small-signal rate equations are two linearly coupled differential equations similar to the voltage and the current equations of a RLC circuit. In a laser cavity the electrons and the photons exchange energy through absorption and emission with the various loss mechanisms dissipating energy in the cavity. Similarly, in a RLC circuit the capacitor and the inductor exchange energy, and the resistor dissipates energy out of the circuit. In addition, the continuity conditions of the electrical circuit (i.e., voltage across the capacitor and current across the inductor are continuous) are equivalent to those of the laser cavity (i.e., the changes in the electron and the photon numbers inside the cavity are continuous if rate equations are being utilized). Motivated by these analogies, we make the assumption that the excited electrons in a laser cavity can be represented with the charge across a capacitor, and the photons in a given mode of the laser with the magnetic flux linkage of an inductor:

$$\delta P(t) = \frac{\Psi(t)}{q(\text{H/s})}, \quad \delta N(t) = \frac{Q(t)}{q}, \quad (4)$$

where division by the electronic charge $q$ and by the fixed units of [Henry per second (H/s)] ensure proper units for the components in the electrical circuit. Assigning electrons to an inductor and photons to a capacitor still would have given us the same results, but the component values in the electrical circuits would turn out to be negative.

We use Eqs. (4) and the standard equations for a capacitor and an inductor (i.e., $Q = CV_C, i_C = dQ/dt$ and $\Psi = Li_L, V_L = d\Psi/dt$) to convert Eqs. (2) into voltage and current equations:

$$v_L(t) = -\Gamma_p Li_L(t) + \sigma_{N\rightarrow p} CV_C(t)(\text{H/s}), \quad (5a)$$
$$i_C(t) = -\Gamma_N CV_C(t) - \frac{\sigma_{p\rightarrow N} Li_L(t)}{(\text{H/s})}. \quad (5b)$$

Equations (5a) and (5b) are the voltage and the current equations, respectively, of the circuit given in Fig. 1 (with a proper choice of component values), and this circuit is exactly the same as the one given in Ref. 7 to model the small-signal behavior of a single-mode laser in the presence of small perturbation. The output light power of the laser $\delta P_{\text{out}}(t)$ is the product of the deviation of the photon number from the equilibrium value $\delta P(t)$, the unit photon energy ($hv$), and the photon-loss rate through the output mirror ($\alpha_m$), so from Eqs. (4) and Fig. 1 we have

$$\delta P_{\text{out}}(t) = \alpha_m \frac{Li_L(t)}{q(\text{H/s})} hv.$$

Similarly, for arbitrary external electrical modulation $[i_m(t)]$, Eq. (2b) is modified as

$$\delta N(t) = -\Gamma_N \delta N(t) - \sigma_{p\rightarrow N} \delta P(t) + \frac{i_m(t)}{q}, \quad (6)$$

such that the current equation (5b) becomes

$$i_C(t) = -\Gamma_N CV_C(t) - \frac{\sigma_{p\rightarrow N} Li_L(t)}{(\text{H/s})} + i_m(t). \quad (7)$$

The circuit is then modified as shown in Fig. 2, and including the electrical parasitics associated with an external current source gives us the circuit published in Ref. 9.

![Fig. 1. Small-signal circuit model of a single-mode laser in the presence of small perturbation:](image-url)
In Eq. (10) we obtain the voltage equation (5a) becomes

\[ v_L = -\Gamma P_L i_L + \sigma_N P C_v(t) \left( \frac{H}{s} \right) \]

\[ + \frac{q(H/s)\eta P_{in}(t)}{h}, \]

where \( \eta \) is the coupling efficiency of the optical input signal into the Fabry–Perot cavity of the laser:

\[ \eta = \frac{T_1(1 - R_2)}{1 + R_1 R_2 - 2(R_1 R_2)^{1/2} \cos \delta}. \]

In Eq. (11) the subscript 1 refers to the input mirror and \( T \) and \( R \) are the transmittance and the reflectance, respectively, of each mirror (with \( T + R + A = 1 \), and \( A \) the mirror absorption). The term \( \delta \) is the mismatch between the wavelength of the input signal and the optical round-trip path length of the cavity. Equation (11) is obtained by slight modification of the procedure given in Ref. 15 in which the transmittance of a passive Fabry–Perot interferometer is calculated. The additional term in Eq. (10) adds to the voltage across the inductor, so it is equivalent to placing a voltage source in series with the inductor–resistor pair in the equivalent electrical circuit (Fig. 3).

4. Multimode Lasers

The model for a multimode laser is more involved. When more than one mode exists in the laser, the rate equations (1) become

\[ \dot{P}_k(t) = G_k P_k(t) - \gamma_k P_k(t) + R_{sp_k}, \]

where \( k \) is the mode number [i.e., Eq. (12a) is actually a set of \( k \) equations for \( k \) modes]. Following the same procedure as for the single-mode laser, we derive the small-signal rate equations [analogous to Eqs. (2)]:

\[ \delta \dot{P}_1(t) = -\Gamma_1 \delta P_1(t) + \sigma_{N-p} \delta N(t), \]

\[ \vdots \]

\[ \delta \dot{P}_k(t) = -\Gamma_k \delta P_k(t) + \sigma_{N-p} \delta N(t), \]

\[ \delta \dot{N}(t) = -\Gamma_N \delta N(t) - \sum_k \sigma_{p-N} \delta P_k(t), \]

where the expressions for the decay rates and the coupling coefficients [i.e., Eqs. (3)] are now given separately for each mode simply by addition of a subscript, from 1 to \( k \), to each of the \( P \) terms.

Using Eqs. (4) in Eqs. (13) along with the standard current–voltage equations for a capacitor and an inductor and rearranging the \( q \) and the Henry per second terms, we obtain one current equation and \( k \) voltage equations (one for each of the \( k \) modes):

\[ v_{L_1}(t) = -\Gamma_1 L_1 i_{L_1}(t) + \sigma_{N-p} C_v(t) \left( \frac{H}{s} \right), \]

\[ \vdots \]

\[ v_{L_k}(t) = -\Gamma_k L_k i_{L_k}(t) + \sigma_{N-p} C_v(t) \left( \frac{H}{s} \right), \]

\[ i_C(t) = -\Gamma_N C_v(t) - \sum_k \sigma_{p-N} L_k i_{L_k}(t) \left( \frac{H}{s} \right), \]

The circuit that satisfies all the above equations is not trivial, and the resulting circuit is shown in Fig. 4 along with the component values. Note that each inductor \( L_{ij} \), which represents an optical mode, is now accompanied by a second inductor \( L_{sj} \) to account for the reduction in the coupling coefficient of that mode with the electrons. The output power of each mode is proportional to only \( L_i \) and not \( L_{sj} \). The dominant mode is given by the left-most inductor–resistor pair.
5. Conclusion

A simple procedure, which was used to model the small-signal behavior of a semiconductor laser as an equivalent electrical circuit, has first been applied to an electrically modulated laser, and the resulting equivalent circuit has been verified to be exactly the same as that given in the literature. The procedure has then been used to model the laser behavior under optical modulation (i.e., when the laser is used as an optical amplifier). Finally, the model has been extended to include multimode operation of the laser. In all cases it has been shown that the voltage and the current equations of the electrical circuits are exactly equivalent to the small-signal rate equations of the semiconductor laser.

The electrical circuits presented in this paper are exact models for a multimode semiconductor laser under external modulation (electrical or optical) as long as the small-signal assumption is not violated (i.e., the modulation range is small compared with the equilibrium values at the given bias condition). Therefore these circuits provide a fast and accurate simulation tool with little computational complexity for small-signal laser behavior. These circuits can further be used in combination with more advanced optoelectronic computer-aided-design programs that use mathematical techniques to obtain the large-signal behavior of a laser through piecing together small-signal behaviors at various regimes.

The authors thank Daniel Hartmann for useful discussions. This study is supported by the U.S. Defense Advanced Research Projects Agency under grant F49620-97-1-0285.

References


Fig. 4. Small-signal circuit model of a multimode laser:

\[
R_n = \left( \frac{H}{s} \right) \frac{\alpha_{N-P_j}}{\Gamma_N}, \quad C = \frac{1}{\left( \frac{H}{s} \right) \alpha_{N-P_j}}, \quad L_j = \left( \frac{H}{s} \right) \frac{\alpha_{P_j-N}}{\alpha_{P_j-N}},
\]

\[
L_{ij} = L_j \left( \frac{\alpha_{N-P_j}}{\alpha_{N-P_i}} - 1 \right), \quad (j = 1, \ldots, k),
\]

\[
R_j = \left( \frac{H}{s} \right) \frac{\alpha_{P_j-N}}{\alpha_{P_j-N}}, \quad (j = 1, \ldots, k),
\]

\[
V_{m}(t) = \frac{q}{S} \left( \frac{H}{s} \right) \frac{\alpha_{N-P_j}}{\alpha_{N-P_j}} \cdot \frac{\alpha_{P_j-N}}{\alpha_{P_j-N}}.
\]

10 September 1998 / Vol. 37, No. 26 / APPLIED OPTICS 6139