End-to-End and Mac-Layer Fair Rate Assignment in Interference Limited Wireless Access Networks

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***Abstract*— In this paper, the problem of end-to-end weighted max-min fair rate assignment in a two-channel multi-hop CDMA wireless access network is discussed. We show that end-to-end weighted global max-min fairness (hierarchical as well as flow-based) can be achieved by simple extension of mac-layer fairness. In particular, we show that weighted end-to-end flow-based as well as hierarchical global max-min fairness can be simply insured if and only if weighted mac-layer max-min and weighted transport-layer max-min fair rates are achieved. The same results can easily be shown to be valid for more general wireless networks, which will be briefly discussed in this paper as well.**

In addition, we discuss a mac-layer algorithm, $MAC-\alpha-G$ algorithm, that, with careful choice of parameters, not only provides weighted α-proportional fairness at the mac layer, but also leads to end-to-end weighted global max-min fairness (both flow-based and hierarchical) with an appropriate higher-layer protocol (i.e. weighted transport-layer max-min fair protocol).

Keywords: System Design, End-to-End Global and Mac layer Max-Min Fairness, Alpha-Proportional Fairness.

I. Introduction and Related Work

Multi-hop wireless access networks, with their easy and cost effective deployment and reconfigurability features are getting attention for many potential applications as the last mile solution. The potential applications of multi-hop wireless access networks include public safety, military and community access networks. In community network projects, a high capacity gateway providing internet connection is located in the neighborhood and the residents are able to reach internet over a multi-hop wireless access network. Similarly, in public safety and military applications, in order to cooperate and coordinate the operations, the first responders and military personnel use wireless access networks.

Multi-hop wireless access mesh networking technology is still to overcome many important challenges to be widely deployed. These challenges range from range and capacity limitation of the wireless links to secure and fair resource allocation. These issues are being addressed and studied by researchers both at industry [11], [12] and academia [15], [13].

Furthermore, leveraging high statistical multiplexing gains in a residential environment, multi-hop wireless and wired access networks are recently introduced for peer-to-peer resource sharing to Internet access such that each individual is able to utilize the fair amount of the peak bandwidth available to the entire community (e.g. neighborhood) [12]. For example, each resident having a broadband access (i.e. DSL) is able to utilize not only his/her own connection bandwidth but also those of his/her neighbors connections over a multihop wireless or wired access network [12].

In [15], authors propose a multi-hop wireless mesh architecture using 802.11 protocol utilizing two wireless cards on each node in the network. In contrast in this paper, we focus on multi-hop CDMA or UWB based wireless access networks. Via power/interference management, we seek to provide fair rate assignment over wireless (multi-access) channels. The literature on mac-layer fairness is rich (see [14], [17], [16], [9], [10], [7]). While [14], [17], [16], [9], [10]) address the fairness issues at the mac layer over a single-hop network, [9] extends this to multi-hop scenario ignoring flow-based end-to-end fairness. Our paper complements these works as it concretizes the relationship between mac-layer fairness and end-to-end fairness. Furthermore, [7] discusses joint rate control and scheduling problem, and [8] examines joint congestion and medium access control both for multi-hop wireless networks in the context of aggregate utility maximization. In both papers, joint problem is shown to be decomposed into two protocol layers and can be solved individually. Our work, on the other hand, specifically discusses that global max-min fair rate assignment problem (via joint transport rate and mac-layer control) decomposes such that it can be solved as independent fair rate assignment problems in each layer.

The main contributions of this paper can be summarized as follows:

End-to-end (flow-based and hierarchical) global weighted max-min fairness can be achieved if and only if both weighted transport-layer and weighted mac-layer max-min fairness are ensured with appropriate weights. (The weights for each link for mac-layer fairness is a function of the weights associated to each flow.)

The remainder of this paper is organized in the following manner: In section 2, the network model is discussed. Section 3 discusses the rate assignment problem and provides the main result of the paper on mac layer vs. end-to-end fairness. In section 4, a weighted mac-layer fair algorithm is discussed. Section 5 includes discussions and examples. Finally, Section 6 concludes the paper.

II. Network Model

In this section, we mainly describe 2-channel CDMA type network model for which the main results of the paper is

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presented. Furthermore, briefly, we discuss a general single-channel network model for which the main results of the paper still apply.

A. Network Model 1

We consider a multi-hop access network which is formed by wireless client devices and the Access Points (APs). Each client associates to one of the APs and APs form a mesh network together. There is also a gateway access point which provides internet connectivity. The clients are able to communicate with each other and access the internet through this access network. We consider a Infrastructure Basic Service Set (IBSS) type architecture which is comprised of an AP and the client devices that associate to that AP. Clients are not able to communicate with each other over a direct link. They first need to send the information to the AP that they associate as in the case of 802.11 IBSS.

Each node in the network (APs and clients) is able to utilize a single transmitter and a receiver. Both transmitter and the receiver can be tuned to 2 non-overlapping channels. Thus each node is assumed to transmit and receive simultaneously over these 2 non-overlapping channels where the inter-channel interference is neglected (Figure 1). The logical connections between a node and its AP or between two adjacent APs is called a link. Let $L = L_1 \bigcup L_2$ denote the set of directional links in the network where L_1 and L_2 are the set of links tuned to channel 1 and 2 respectively. Any link $l \in L$ can also be represented by the transmitter node i and the receiver node j such that $l = (i, j)$. As the case with CDMA networks, each link is given a code. In other words, the links tuned to the same channel have the ability to be active simultaneously.

In this paper, we assume that all nodes in the network are able to hear each other.

Each AP is considered to be a wireless bridge such that the packets are forwarded in layer 2 throughout the entire access network. Spanning tree protocol is used [4] to form a loop free topology where the learning bridge algorithm works well [4]. There are many recent papers considering loop free topologies [15] for multi-hop access networks.

In this work, we also consider the capacity of each wireless link as a linear function of the related signal to interference noise ratio (SINR)(e.g. low signal to noise ratio regime).

$$X_{i,j} = B\frac{P_{i,j}G_{i,j}}{\sum_{m,n\neq i,j} P_{m,n}G_{m,j} + \gamma} \tag{1}$$

where $X_{i,j}$ is the capacity of link i, j and i and j are the end nodes of the link. B is the bandwidth allocated for the related channel, $P_{i,j}$ is the power transmitted on link i, j and $G_{i,j}$ is the attenuation constant such that $P_{i,j}G_{i,j}$ is the received power at the receiving end of link i, j. γ is the ambient noise.

Each link i, j has a power budget such that

$$0 \leq P_{i,j} \leq Pmax_{i,j} \tag{2}$$

On the other hand, a flow is defined to be a logical connection between any mobile client device and the gateway, or between any two mobile client devices. Let F denote the set of all flows in the network. We assume that routing is given by a fixed matrix $\Psi = [\Psi_{p,l}]_{|F|\times|L|}$, $\Psi_{p,l} = 1$ if $p \in F_{l=(i,j)}$ otherwise it is 0 where $F_{i,j}$ denotes the set of flows traversing link (i, j). In this paper, due to the loop free nature of the topology it is indeed a fact rather than an assumption.

The rate of flow p, R_p, is the information rate that the related source node conveys to the destination node.

The rate of link $l = (i, j)$ should be greater than or equal to the aggregate rate of flows that are traversing the link such that

$$T_{i,j} = \sum_{p\in F_{i,j}} R_p \leq X_{i,j}, \quad R_p \geq 0 \quad \forall p \in F \tag{3}$$

We assume a transport layer protocol, given the link rates, that assigns the rates among the end-to-end flows. It is assumed that each flow has infinite demand. Independent of the end-to-end flow rates, we also assume a mac-layer protocol that sets the link capacities with respect to the equation 1.

B. Network Model 2

In this model, we assume a multi-hop wireless access network where still each node is able to hear each other. In other words, each link interfere with all other links in the access network. There exist a single channel that is used by all the links in the network. Routing or bridging or any other path setup mechanism is assumed. The capacity of each link $l = (i, j)$, $X_{l=(i,j)}$, is assumed to be a strictly increasing function of the average transmitted power $ATP_{i,j}$ which is equal to $P_{i,j} \times S_{i,j}$ where $S_{i,j}$ is roughly the frequency of the link i, j being used. The notion of average transmitted power can describe perfectly scheduled networks as in [6] or any other mechanism like 802.11 or a CDMA type multi-hop network. On the other hand, the link rate $X_{i,j}$ is assumed to be a strictly decreasing function of the average transmitted powers of all other links (e.g. $ATP_{m,n}$ $\forall$ $(m, n) \neq (i, j)$).

The feasibility region for flow rates is the same as in network model 1.

III. Rate Assignment Problem

In this section, end-to-end global (flow based as well as hierarchical) and mac-layer rate assignment policies and the relationship between them are discussed.

A. Definitions

First, we introduce the following definitions.

Definition 1: A vector of rates R is **weighted max-min fair** with weight vector W,if it is feasible and for each flow i, the rate of flow i, R_i, can not be increased while maintaining feasibility without decreasing R_j for some flow j for which $R_jW_j \leq R_iW_i$. As a special case, for $W_i = 1$ $\forall i$, vector R is said to be max-min fair [3].

Definition 2: A feasible vector of link rates, X is said to be **(weighted) mac-layer max-min fair** if the link rate vector, X, belongs to Y defined as:

$$Y = \{(.., X_{i,j}, ..) : X_{i,j} = B\frac{P_{i,j}G_{i,j}}{\sum_{m,n \neq i,j} P_{m,n}G_{m,j} + \gamma},$$
$$0 \leq P_{i,j} \leq Pmax_{i,j}\} \quad (4)$$

and is (weighted) max-min fair.

Definition 3: A vector of end-to-end flow rates, R, is said to be **(weighted) transport layer max-min fair**, given fixed routing matrix Ψ and link rate vector, X, if R belongs to V_X as defined:

$$V_X = \{(.., R_p, ..) : \sum_{p \in F_{i,j}} R_p \leq X_{i,j} \quad \forall (i,j) \in L,$$
$$R_p \geq 0 \quad \forall p \in F\} \quad (5)$$

and is (weighted) max-min fair.

Definition 4: Given a routing matrix, Ψ, a vector of end-to-end flow rates is said to be **end-to-end global flow-based max-min fair**, if the flow rates are chosen from set S: $S = \bigcup_{\forall X \in Y} V_X$ and is max-min fair.

Unlike the transport-layer fairness, in global fairness the link rates are not assumed to be given. In our wireless access network the capacity of a link is a function of the other link capacities, therefore in order to enforce the global fairness we need to compute both the link rates and the flow rates. A numeric example is available in [1].

Definition 5: Given a feasible vector of flow rates, R, we say that link $l = (i,j)$ is a **weighted bottleneck link** with weight vector W with respect to R for a flow p traversing link l , if $T_{i,j} = X_{i,j}$ and $R_pW_p \geq R_qW_q$ for all the flows q traversing link l. If all the weights are equal to 1 then link l is said to be a **bottleneck link** as in [3].

B. Weighted End-to-End Flow-Based Global Max-Min Fairness: Main Result

In this section, we establish a relationship between mac-layer and end-to-end global fair rate assignments in the discussed access network. We show weighted fairness at the mac-layer and weighted max-min fairness in the transport layer ensure end-to-end global weighted max-min fairness per flow, vice versa. To prove this we need the following facts.

Fact 1: Weighted mac-layer max-min fair rate assignment in our network with weight vector W and rate vector X assigns the rates to each link tuned to the same channel such that $X_lW_l = X_dW_d \; \forall l, d \in L_i : i = 1, 2$. Proof can be found in [1]. Note that as a special case mac-layer max-min fairness ($W_l = 1$, $\forall l$) assigns the same rate to all the links [20].

Fact 2: A vector of link rates X is weighted mac-layer max-min fair with weight vector W if it is achievable (i.e. in the capacity region) and it is the maximal among vectors E such that $W_lE_l = W_dE_d$ for all $\forall l, d \in L_i : i = 1, 2$. (Similarly, $E_l = E_d$ for max-min fair case.) (A vector, V, is maximal when there is no other vector, D, of which elements are not less than those of V and at least one is strictly greater). Proof of this fact is a direct result of Fact1 and the definition of (weighted) max-min fairness.

Fact 3: End-to-end flow-based global weighted max-min fair rate assignment with weight vector W and flow rate vector R assigns the flow rates such that $R_iW_i = R_jW_j \; \forall i, j \in F$. Note that if all the weights are the same then the rate assignment in question assigns the same rates to each flow in our access network. Also note that each flow in our network utilizes both the channels. The proof is available in [1]. The proof for all the weights equal to 1 has the same logic as in [20].

Fact 4: If a feasible flow rate vector, R, is said to be end-to-end global flow-based weighted max-min fair with weight vector W then there exists a channel, i, such that with respect to R, each flow has a weighted bottleneck link, l (with W), tuned to channel i and each such link l (i.e $\forall l \in Li$) is a weighted bottleneck link (with W) for some flow in the network and the link rate vector of channel i is maximal. Proof is available in [1].

Now we provide the main result of this section.

Let W be the weight vector with which R (the flow rate vector) is end-to-end global flow-based weighted max-min fair.

Let N denote a vector such that $N = ((n_1)^{-1}, ..., (n_l)^{-1}, ...(n_{|L_b|})^{-1})$ where $n_l = \sum_{f \in F_{l=(i,j)}} (W_f)^{-1}$ and $F_{i,j}$ denotes the set of flows traversing link (i,j) and $|L_b|$ is the number of bottleneck links.

Theorem 1: In our access network, end-to-end flow-based global weighted max-min fair rate allocation, R, with weight vector W can be achieved if and only if transport-layer weighted max-min fairness with weight vector W and the weighted mac-layer max-min fairness with weight vector N (among the resulting bottleneck links with respect to R) are ensured.

Proof: We first show that end-to-end flow-based global weighted max-min fairness with weight vector W leads to weighted mac-layer max-min fairness with weight vector N and transport-layer weighted max-min fairness with weight vector W. Note that throughout the proof, the links discussed are the weighted bottleneck links with weight vector W as defined in Definition 5.

First, we can easily show that if a vector of flow rates, R is end-to-end global flow-based weighted max-min fair with W then it is also transport-layer weighted max-min fair with W. The proof is so simple by contradiction. Assume that the vector, R is global weighted max-min fair but not transport-layer weighted max-min fair with the same weight vector. Then there should exist a flow i, of which rate R_i can be increased without decreasing R_j for some flow j for which $W_iR_i \geq W_jR_j$, which is also a contradiction to the end-to-end global flow-based max-min fairness.

Next, by contradiction assume that the vector of end-to-end flow rates, R, is both end-to-end global weighted max-min fair and transport-layer weighted max-min fair with W but the corresponding link capacity vector, X, is not weighted max-min fair with weights N.

End-to-end global flow-based weighted max-min fair rate assignment assigns rates to each flow inversely proportional to their weights that is $W_qR_q = W_pR_p = r \quad \forall p, q \in F$

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(see Fact3). In the presence of end-to-end weighted max-min fairness, the resulting aggregate flow rate traversing link $l = (i,j)$, T_l, will be $T_l = \sum_{p \in F_l} R_p = \sum_{p \in F_l} (W_p)^{-1} r = r \times n_l$. Since we are discussing on the bottleneck links then $T_l = X_l = r \times n_l \; \forall l \in B$.

It can easily be seen that the link capacities satisfy the condition in Fact 2 for weighted fairness such that $X_{i,j} N_{i,j} = X_{a,b} N_{a,b}$ where $N_{i,j}$ denotes the element of vector N corresponding to link i,j.

However, by contradiction we have assumed that the vector X is not weighted max-min fair with weight N, then according to Fact 2, vector X can not be maximal, otherwise it would be the weighted max-min fair rate.

Therefore, one can increase all the link rates in the network and this increase can be easily mapped into an increase in all the end-to-end flow rates which contradicts with the end-to-end global weighted max-min fair rate assumption that we had in the beginning. Thus, we have shown so far that if a vector of flow rates is end-to-end flow-based global weighted max-min fair with W then it is also transport-layer weighted max-min fair with W and the corresponding vector of link rates is weighted mac-layer max-min fair with weight vector N.

Conversely, assume that the link rate vector X is weighted max-min fair with weights N and assume transport layer weighted max-min fairness with weight vector W, then all the flow rates will be assigned inversely proportional to their weights. By contradiction assume that the end-to-end flow rate vector R is not weighted max-min fair with W globally. Then we should be able to increase all the R_is without decreasing any of them (since we know that end-to-end flow-based global max-min fairness indeed assigns rates to all the flows inversely proportional to the corresponding weights). This actually requires an increase in all the link rates inversely proportional to their link weights which contradicts with the weighted mac-layer max-min fair assumption. ■

(This theorem is valid for both network model 1 and network model 2.)

This result is interesting in the sense that there is small interaction between the mac layer and the transport layer. The only information that mac layer needs to know is the sum of the inverse weights of the flows that are passing through (For $W_i = 1 \forall i$, that is the max-min fair case, the only information to be passed to the mac-layer is the number of flows passing through). Then the weighted max-min mac-layer scheme with appropriate weights and with a weighted max-min fair transport protocol leads to a weighted fair flow rate allocation in the end-to-end and global manner.

In the next section, we discuss hierarchical global weighted max-min fairness which can be achieved by end-to-end flow-based global weighted max-min fairness with appropriate choice of the system parameters.

C. End-to-End Hierarchical Weighted Max-Min Fair Rate Assignment

Although flow based weighted max-min fairness is the classical way of studying the fairness problem, in real world different fairness variations may appear.

Considering a community network application as in [12] where each IBSS belongs to a resident, each resident participating the access network would like to have a fair share of the overall bandwidth which is proportional to what they pay for their internet access speed. In an access network where end-to-end flow based weighted max-min fairness is enforced (with all weights equal to 1), a resident utilizing higher number of connections will have a higher share of the overall bandwidth with respect to the ones having smaller number of connections. Enforcing hierarchical fairness each resident is ensured to have a fair share of the network bandwidth first and then within the same IBSS the fairness among the flows can be enforced.

Therefore, in our network, one of the interesting fairness criterion may be a hierarchical fairness such that fairness is first ensured among the Infrastructure Basic Servive Sets (IBSS) (i.e among the set of flows utilized by different IBSSs) and then among the individual flows in the same IBSS.

Definition 6: Let M_a be the set of flows belonging to subgroup a such that $\bigcup_a M_a = F$ and $M_a \bigcap M_b = \emptyset$ $\forall a, b : a \neq b$. Let D be the rate vector where the ath element denotes the aggregate information rate of subgroup $a : D_a = \sum_{i \in M_a} R_i$, and let Z be the corresponding weight vector for the subgroups. Lastly, let H_a denote the vector of rates of the individual flows belonging to M_a and V_a denotes the weight vector for the flows in M_a.

A vector of flow rates, R, is said to be **weighted hierarchical max-min fair with a weight vector Z, and a vector of vectors** $V = (V_1, V_2, ...V_T)$, where T is the number of subgroups, if first the vector, D, is weighted max-min fair with weight vector Z and the rate vector for flows in each subgroup a, H_a, is weighted max-min fair with weight vector V_a.

More formally, a vector, R, is said to be **weighted hierarchical max-min fair with a weight vector Z, and a vector of vectors** $V = (V_1, V_2, ...V_T)$, if it is feasible and if for each flow, $i \in M_a$, the rate, R_i, can not be increased, while maintaining feasibility without decreasing R_j for some flow $j \in M_b$ for which $V_{b,j} R_j \leq V_{a,i} R_i$ when $a = b$ and $Z_b D_b \leq Z_a D_a$ when $a \neq b$. When all the weights are all equal to 1, the rate vector R is called Hierarchical Max-Min fair as similarly defined in [13].

Fact 5: Using similar arguments for mac-layer and end-to-end flow based max-min fair rate assignment policies, it can be claimed that such a hierarchical fairness policy leads to the aggregate flow rates for each IBSS such that $Z_a D_a = Z_b D_b$ $\forall a, b$. Again using the same argument, we can claim that each flow belonging to the same IBSS have rate as follows $V_{a,i} R_i = V_{a,j} R_j$ $\forall a$ and $\forall i \in M_a$. Proof is available in [1].

Fact 6: A vector of flow rates, R is hierarchical weighted max-min fair with Z and V as defined above, if it is achievable (i.e. in the capacity region) and corresponding D vector is the

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maximal among vectors, E, such that $Z_a E_a = Z_b E_b \ \forall a, b$ and given the D vector, for each subgroup a, the related vector H_a is the maximal among the vectors, B^a, such that $V_{a,i} B_i^a = V_{a,j} B_j^a \ \forall i, j \in M_a$. (It is a direct result from the previous fact on hierarchical fairness.)

Let Q be the weighted rate of each subgroup (e.g. resident) utilizes such that $Q = Z_a D_a \ \forall a$ where the weighted hierarchical fairness policy with Z and V is enforced. Then the aggregate rate that the subgroup a utilizes will be $\frac{Q}{Z_a}$, whereas the rate of flow i, R_i in the same subgroup will be $R_i = \frac{V_{a,j} R_j}{V_{a,i}} \forall i$.

From the definition of D_a,
$D_a = \sum_{j \in M_a} R_j = \sum_{i \in M_a} \frac{V_{a,j} R_j}{V_{a,i}} = \frac{Q}{Z_a}$.

Therefore, rate of each flow $i \in M_a$ can be written as
$R_i = \frac{Q}{Z_a V_{a,i} (\sum_i \frac{1}{V_{a,i}})}$

Let $s(i)$ denote the subgroup to which ith flow belongs and let $K_{a,i} = Z_a V_{a,i} (\sum_i \frac{1}{V_{a,i}})$.

Let W be the vector such that
$W = ((K_{s(1),1}), (K_{s(2),2}), (K_{s(3),3}), ...(K_{s(|F|),|F|}))$
where $|F|$ is the number of all flows.

(Considering the above discussion, we can safely say that any weighted hierarchical max-min fair vector of flow rates, R, satisfies the condition $R_i W_i = R_j W_j$.)

Theorem 2: End-to-end hierarchical global weighted max-min fairness with Z and V can be achieved if and only if end-to-end flow-based global weighted max-min fairness with weight vector W is ensured. Outline of the proof can be found in [1].

Corollary: Using the results above and Theorem 1, it can easily be seen that a vector of flow rates, R is said to be hierarchical weighted max-min fair with Z and V, if and only if transport layer weighted max-min fair with weight vector W (as defined above) and weighted mac-layer max-min fairness with weight vector N are achieved, where the lth element of vector N, N_l, equals $(\sum_{f \in F_l} (W_f)^{-1})^{-1}$.

In the next section, we would like to propose a mac-layer scheme that is able to achieve the weighted mac-layer max-min fairness we are looking for.

IV. Mac-Layer Algorithm Ensuring End-to-End Global and Mac-Layer Fairness

In this section, we discuss a mac-layer algorithm that enforces mac-layer weighted α-proportional and mac-layer weighted max-min fair rate assignments.

As we discuss in the previous sections (i.e. Theorems 1 and 2) with appropriate weights, mac-layer weighted max-min fair rate assignment results in link capacities such that with appropriate higher layer mechanisms (i.e. (weighted) max-min transport layer protocol), end-to-end (flow based and hierarchical) max-min fairness is achieved globally. In addition to this, considering Fact 4, that is, all the links tuned to a single channel being the bottleneck links for all the flows in the network, all we need to have is a mac-layer scheme that ensures the weighted mac-layer fairness among the links tuned to the same channel.

In this section, we begin with mac-layer weighted α-proportional fair rate assignment. Next, we introduce a mac-layer algorithm, $MAC-\alpha-G$, which is a general mac-layer algorithm where with appropriate choice of parameters α and G, not only mac-layer α-proportional fairness but also end-to-end max-min (flow based and hierarchical) fairness can be achieved.

A. Mac-layer Weighted Max-Min Fair Rate Assignment (End-to-End Global Fair Rate Assignment)

In this section, we discuss an approach to compute the weighted max-min fair link rates in a distributed manner.

Definition 8: Similar to [19], we consider a generalization of proportional fairness by considering a different utility function for each entity. A vector of rates, R, is **α-proportional fair** if it maximizes the sum of the utilities when the utility function for entity i is $-(-ln(R_i))^\alpha$.

1) Weighted α-Proportional Fairness Approach:: The overall optimization problem for weighted max-min fairness is

$$\textbf{Maximize} \quad \sum_{i,j} U(X_{i,j}) = \sum_{i,j} -\frac{1}{g_{i,j}} (-ln(g_{i,j} X_{i,j}))^\alpha \tag{6}$$

$$\textbf{Subject To:} \quad 0 \leq P_{i,j} \leq P_{max} \tag{7}$$

where $X_{i,j}$ is defined in equation 1 and $g_{i,j}$ is a weight associated to the link (i, j) to ensure the weighted fairness.

CLAIM: The system trying to maximize the aggregate utility function $U = \sum_{i,j} U(X_{i,j})$ leads to weighted α-Proportional fair rate assignment. Weights equal to 1 and $\alpha = 1$ corresponds to mac-layer proportional fair case whereas as α goes to infinity the system converges to weighted max-min fair rate assignment. The proof of this claim can be found in [1].

The above problem is not a convex programming problem (CPP). Using the change of variable $P_{a,b} = e^{Z_{a,b}}$, the problem can be converted into a CPP such that $Y_{i,j} = Z_{i,j} + ln(g_{i,j} WG_{i,j}) - ln(\sum_k (e^{Z_{m,n}} G(m, j)) + \gamma)$ is a concave function. (log-sum-exp function is a convex function [2] and an affine composition of any such function is also convex [2]). The constraint part stays also convex such as $0 \leq e^{Z_{a,b}} \leq P_{max}$.

In addition to that $\frac{1}{g_{i,j}} (-Y_{i,j})^\alpha$ can be shown to be a convex function of the logarithmic transmitted power strengths (e.g. $Z_{i,j} = ln(P_{i,j})$) where $0 \leq g_{i,j} X_{i,j} \leq 1$ and $\alpha \geq 1$. The proof can be found in [1].

We use Gradient Projection method [5] to solve the above optimization problem such that

$$Z_{i,j}^{n+1} = [Z_{i,j}^n + \theta^n \frac{\partial U}{\partial Z_{i,j}}]^+ \tag{8}$$

where n denotes the iteration number and U is the sum of all utility functions and $[f]^+$ denotes projection on the set

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$0 \le P_{i,j} \le P_{i,j}^{max} \forall i$ which is equal to $min(max(0, f), P_{i,j}^{max})$ and

$$\frac{\partial U}{\partial Z_{a,b}} = \alpha(\frac{1}{g_{a,b}})(-ln(g_{a,b}X(a,b)))^{\alpha-1} - \sum_{k,l \ne a,b} \alpha \frac{e^{Z_{a,b}} G_{a,l}}{\sum_{q,r \ne k,l} e^{Z_{q,r}} G_{q,l} + \gamma} (\frac{1}{g_{k,l}})(-ln(g_{k,l}X_{k,l}))^{\alpha-1} \quad (9)$$

$$f_{a,b} = 1 - \sum_{k,l \ne a,b} \frac{e^{Z_{a,b}} G_{a,l}}{\sum_{q,r \ne k,l} e^{Z_{q,r}} G_{q,l} + \gamma} (\frac{g_{a,b}}{g_{k,l}})(\frac{ln(g_{k,l}X_{k,l})}{ln(g_{a,b}X_{a,b})})^{\alpha-1} \quad (10)$$

we can say that the equation 9 becomes negative when $f_{a,b} < 0$ or vice versa.

The above iterative algorithm based on gradient projection method corresponds to a distributed algorithm that targets the weighted α-proportional fair rates (e.g. asymptotically weighted max-min fair rates). Here we present a mac-layer algorithm where the increase and decrease coefficients are assumed to be small enough for convergence. The decision whether to increase or decrease the power is based on the gradient projection method. Having the coefficients small enough, gradient projection method is shown to converge to one of the stationary points (the point where the gradient is zero) [5]. Here since the problem is a CPP it has only a single stationary point.

As a feedback mechanism we assume that each link in the network advertises its weight, $g_{i,j}$, its current capacity and the power it receives from all other links that it is able to hear.

Given any arbitrary initial power vector, P, at each iteration n, each link (a,b) computes the value of $f_{a,b}(P)^n$ and increases its transmit power, P_k^n, if $f_{a,b}(P)^n$ is positive or decrease if it is negative.

The whole power control scheme which we call MAC-$\alpha - G$ for link a, b can be written as follows.

MAC-$\alpha - G$ Algorithm
STEP1: Initially start with a random power dissipation at each link.
STEP2: At iteration n

IF $f_{a,b}^n > 0$ (or a positive threshold) then
Increase $P_{a,b}$ (e.g. $P_{a,b}^{n+1} = P_{a,b}^n + \beta$)
ELSE IF $f_{a,b}^n < 0$ (or a negative threshold) then
Decrease $P_{a,b}$ (e.g. $P_{a,b}^{n+1} = P_{a,b}^n - q$)
ELSE Do not change $P_{a,b}$ (e.g. $P_{a,b}^{n+1} = P_{a,b}^n$)

where β and q are appropriate constants.

As mentioned previously, the mac-layer algorithm (i.e MAC-$\alpha - G$) (providing mac-layer max-min fairness with weight vector G) can be used with a weighted transport layer protocol to ensure end to end global max-min fairness.

An alternative problem formulation and algorithm is discussed in [1].

V. Discussions and Examples

In this section, we consider a single gateway access network where there are 3 Access Points and 5 client devices, as in Figure 1.

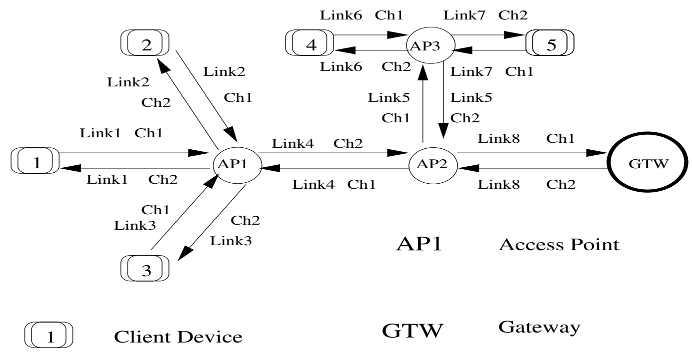

Fig. 1. First example network

The attenuation constant is modelled as $G_{i,j} = d_{i,j}^{-n}$ where $d_{i,j}$ is the distance between nodes i and j and n is assumed to be 2. Link capacities presented are normalized between zero and one unit of capacity.

We first consider the following scenario where each link $(l = (i,j))$ is assumed to have infinite amount of traffic to transmit from the transmitter node i to the receiver node j all the time. Each of the links are assumed to run the MIMD-$\alpha - G$ algorithm where all the weights and α are all equal to 1 (mac-layer fairness scenario).

The resulting rates of each link tuned to channel 1 are illustrated as the value of α increases from 1 to 60 in Figures 2 and 3 respectively. As can be seen in the figures as the value of α increases the capacities on each link converges to the same value which is consistent with the max-min fair rate assignment fact (i.e. Fact 1).

Next, we consider a traffic scenario where the wireless clients number 1,2,3 and 5 has infinite traffic demand to the outside world through the gateway node. Here the end-to-end flow-based global max-min fair rate allocation and the end-to-end hierarchical global fairness are examined.

Consider the end-to-end flow-based case: The link weight, $g_{l=(i,j)}$ for a link l is as follows (1 over the number of flows traversing): For Channel 1, $g_1 = 1$; $g_2 = 1$; $g_3 = 1$; $g_7 = 1$; $g_8 = 1/4$; and for Channel 2, $g_4 = 1/3$; $g_5 = 1$;. Figure 4 and 5 show the MAC-$\alpha - G$ allocations for large α ($\alpha = 60$) such that link rates converge to the weighted max-min fair rates in the mac layer with the weights given above. The resulting link capacities are available in Figure 4 for channel 1 and in Figure 5 for channel 2 respectively. As can be seen, the capacities assigned on each link is approximately inversely proportional to the weights assigned to that link (i.e. for channel 1 X=[0.0192,0.0196,0.0197,0.0216,0.0848] and for channel 2 X=[0.0992,0.0317]) which is consistent with Fact1. In this case, channel 1 includes the bottleneck links which assigns lower rate (0.02 unit capacity) to a single flow than channel 2 does (0.03 unit capacity) in the presence of a max-min fair transport protocol.

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Here we see the need for hierarchical fairness. If such a network is deployed in a community network where each AP is located in a residence, then flow based max-min fair rate assignment assigns to residence 1, 3 times the bandwidth (0.06 unit capacity) it assigns to residence 3 (0.02 unit capacity). This is not desirable for such applications.

Instead, hierarchical fairness model can be more appropriate to consider, where each residence or AP has the fair share of the network capacity.

In the case of hierarchical max-min fair rate assignment, we have the following link weights. For Channel 1, $g_1 = 3$; $g_2 = 3$; $g_3 = 3$; $g_7 = 1$; $g_8 = 1/2$. For Channel 2, $g_4 = 1$; $g_5 = 1$; In Figures 6 and 7 the corresponding rates for links tuned to channel 1 and channel 2 are available respectively. Each IBSS (or AP) is given around 0.0450 unit capacity, where each flow of IBSS1 (or AP1) gets around 0.0150 unit capacity while single flow of IBSS3 (or AP3) gets 0.0450 unit capacity by itself. Channel 1 has all the weighted bottleneck links also in this case, whereas channel 2 offers slightly higher link rates (i.e. for channel 1 X=[0.0153,0.0147,0.0148,0.0477,0.0938] and for channel 2 X=[0.0544,0.0545]).

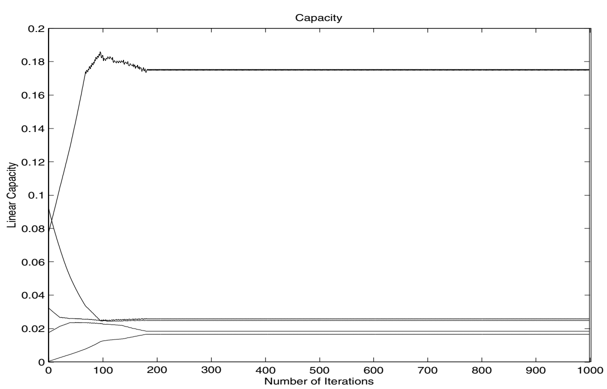

Fig. 2. Capacity, mac layer, alpha =1, channel 1

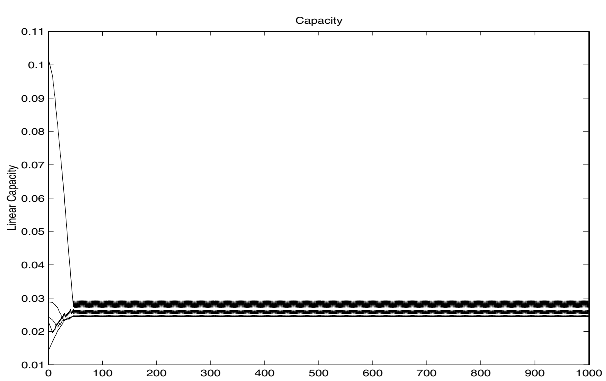

Fig. 3. Capacity, mac layer, alpha = 60, channel 1

In the next example, the flow rate vector, R, is required to be weighted hierarchical max-min fair with weight vectors $Z = [2, 3]$ and $V_1 = [1, 2, 3]$. V_2 can take any value since there is only one flow in that residence. Using Theorem 2, R

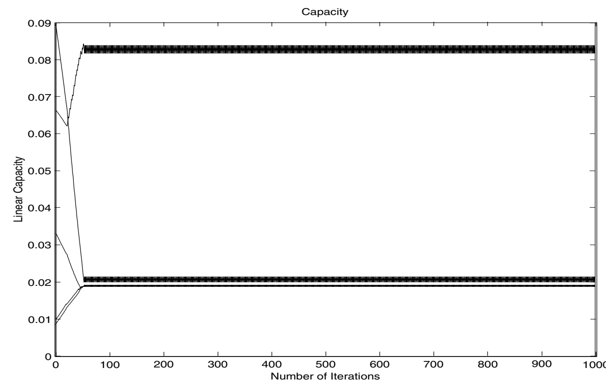

Fig. 4. Capacity, e2e flow based, alpha =60, channel 1

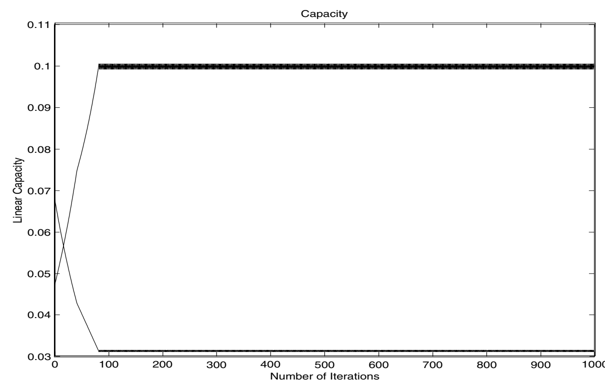

Fig. 5. Capacity, e2e flow based, alpha = 60, channel 2

is also end-to-end weighted max-min fair with weight vector W which is equal to $W = [11/3, 22/3, 11, 3]$. Considering theorem 1, the link rate vector, X, is weighted mac-layer max-min fair with weight vector N. In this example, $N = [11/3, 22/3, 11, 3, 6/5]$ for the links tuned to channel 1 and $N = [2, 3]$ for the links tuned to channel 2. Figures 8 and 9 indicate the resulting rates of each link tuned to channel 1 and 2 respectively (when each link run the MAC-$\alpha - G$ algorithm). The link rate vector, X, turns out to be $X = [0.0288, 0.0144, 0.0096, 0.0379, 0.0908]$ for the links on channel 1 and $X = [0.0652, 0.0412]$ for the links on channel 2. Assuming a weighted transport layer max-min fair protocol with weight vector W, the links on channel 1 becomes the bottleneck link as described in Fact 4. Then the flow rate vector R becomes $R = [0.0288, 0.0144, 0.0096, 0.0379]$. Furthermore, the rate vector, D, denoting the aggregate rate utilized by each IBSS (or residence) becomes $D = [0.0528, 0.0379]$. As can be seen the elements of both vectors X, D and R are almost inversely proportional to the elements of the vectors N, Z and W respectively as stated in Facts 1, 3 and 5.

As can be seen in these examples, using appropriate transport layer (weighted max-min) protocols and the mac-layer (weighted max-min) protocols, fair resource allocation (weighted max-min fair) can be ensured globally not only among individual flows but also among the residents that may utilize several number of flows.

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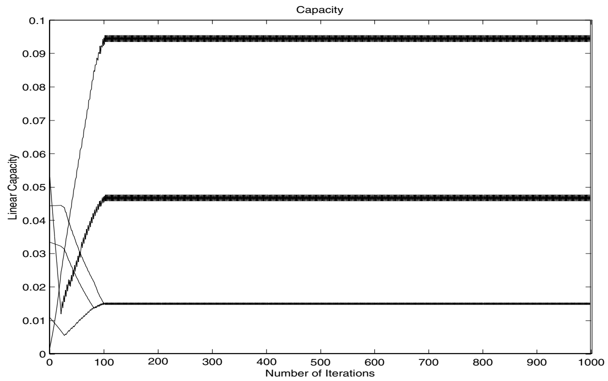

Fig. 6. Capacity, hierarchical, alpha = 60, channel 1

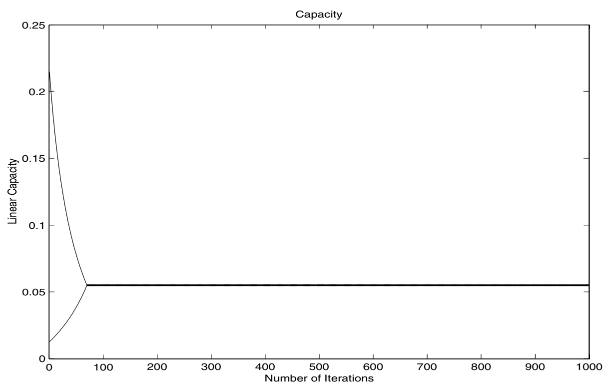

Fig. 7. Capacity, hierarchical, alpha = 60, channel 2

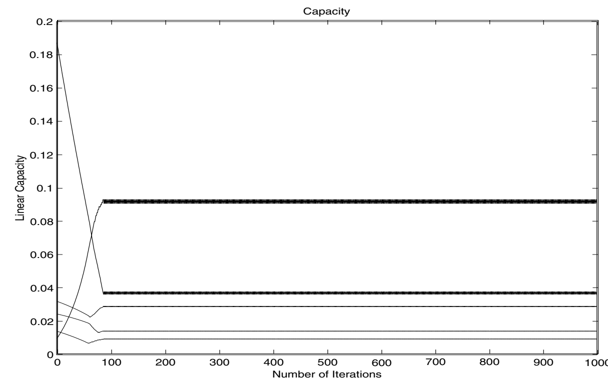

Fig. 8. Capacity, e2e weighted hierarchical based, alpha =60, channel 1

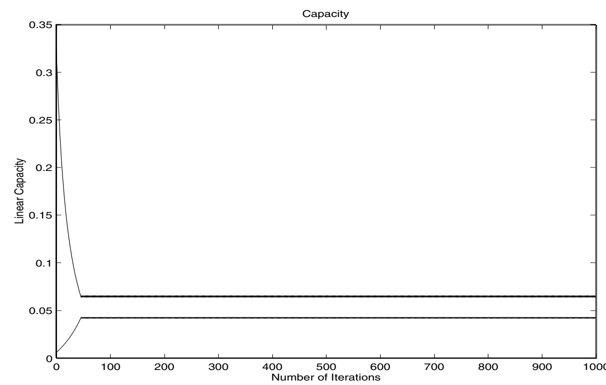

Fig. 9. Capacity, e2e weighted hierarchical based, alpha =60, channel 2

VI. Conclusions

In this paper, we show that in our wireless access network, end-to-end global fairness can be achieved via enforcing weighted mac-layer fairness. Particularly, we show that end-to-end global (flow-based and hierarchical) weighted flow-based max-min fairness is achieved if and only if transport-layer weighted max-min and weighted mac-layer max-min fair rate assignments are ensured. This result suggests that by designing intelligent mac-layer schemes, one can ensure end-to-end global fairness while requiring small interaction among layers. Furthermore, we propose a mac-layer algorithm to achieve weighted mac-layer fairness. Needless to say that such mac-layer algorithms in conjunction with Theorem 1 and 2 can be used to achieve end-to-end global fairness.

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