Leveraging Downlink for Optimal Uplink Rate Allocation: An Incentive Compatible Approach

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***Abstract*— In this paper, we consider joint uplink and downlink rate allocations over a multicast/broadcast pair in a single-cell system. We assume a network of heterogeneous users, where the uplink utility of each user is held as private information and is unknown to the base station. The challenge is to design an optimal rate allocation scheme in the presence of such incomplete information. Here, we provide an incentive-compatible mechanism that leverages downlink demand to ensure that users truthfully reveal their uplink utilities, resulting in a socially optimal uplink rate allocation.**

I. Introduction

As the demand for broadband wireless data services grows, it becomes increasingly necessary to re-examine system design with respect to resource allocation. Wireless spectrum is an inherently shared and limited resource, making efficient resource allocation a crucial aspect of any network design. One of the difficulties of this is the fact that the definition of what constitutes an efficient resource allocation often depends on private information held by the end-users. For example, we may be interested in minimizing delay and queue backlogs in the system, but in an uplink scenario the queue backlog is private information known only to the user. This problem is even more pronounced in the case of QoS delivery in networks where data, voice, and video traffic coexist. Similarly, we may be interested in maximizing the aggregate utility of the users, or we may be interested in a trade-off between fairness and priority for different users while users hold key information needed to establish a “socially optimal” solution. The common thread in all of these scenarios is the fact that the network needs to know information about the end-users in order to determine whether a particular rate allocation is efficient.

In a scenario where users hold private information but are well-behaved (meaning they are interested in helping the network achieve an efficient allocation), then the network can simply ask the users for their private information. However, in realistic scenarios where users are selfish and interested in maximizing their own utility (known as strategic users), users may have an interest in misrepresenting their private information for their own benefit, even at the expense of overall network efficiency [1], [2]. Thus, we need to develop a mechanism that creates an incentive for truth telling, ensuring that users have interest to truthfully reveal their private information.

The concept of incentive compatible mechanism design has long been studied, and applies to a wide array of social utility problems. In economics literature, an elegant way to address this problem is the introduction of a *numeraire commodity* - a commodity for which users have extremely large or infinite demand (e.g. money). Users are charged (in terms of this numeraire) for their consumption of the original commodity of interest in such a way that their overall utility is maximized when they behave in a socially responsible manner [3]. The application of these ideas to resource allocation for communication networks is not a new one. A common way to address issues of incentive compatibility in both wired and wireless networks is to use dollar-valued pricing schemes in which money is modeled as the numeraire commodity and users pay for service (see e.g. [4], [5], [6], [7], [8]).

The goal of our paper is to develop similar mechanisms where the numeraire commodity is the downlink rates. We use the inherent asymmetry between uplink and downlink (in terms of control and demand) to model the downlink bandwidth as a numeraire commodity. In particular, we use a centralized downlink scheduler to construct an incentive compatible mechanism to ensure a socially efficient use of the uplink. We show that with an appropriately designed downlink scheduler the socially optimal uplink rate allocation emerges as a dominant strategy for all users. To the best of our knowledge, this is a novel approach which has not previously been studied in the context of cellular communication.

The rest of this paper is organized as follows. Section II introduces the notation and problem formulation, while Section III introduces the joint rate allocation mechanism which is the focus of this paper. Finally, Section IV gives our conclusions and areas of future work.

II. Socially Optimal Rate Allocations

In this paper, we consider uplink and downlink rate assignments for a single-cell system. There are a total of N users in the system, each of whom communicate to a single base station over bandwidth W. Each mobile transmits over the air to the base station with uplink rate α_i, and available uplink transmit power p_i. The base station transmits over the air to each mobile with downlink rate A_i, and available downlink transmit power P. The uplink and downlink channel gains are symmetric, and denoted by G_i, and the thermal noise level is N_0.

A. Uplink Rate Allocations

In order to define the notion of a socially optimal uplink rate allocation, we must first define the notion of a feasible rate region. In any wireless system, the definition of what constitutes a feasible rate-power pair is highly dependent upon both the application and design of a particular system. Constraints may include minimum or maximum power constraints, interference limits, minimum rate guarantees, QoS metrics, or delay requirements. Here, we work with information-theoretic notions of capacity. Specifically, we work with the standard multiple access region in an AWGN channel, defined below.

Definition 1: The *uplink feasible rate region* Δ_U is the set of rates $\underline{\alpha}$ that satisfy the following conditions:

C1. $\alpha_i \geq 0 \ \forall \ i \in \mathcal{N}$

C2. $\sum_{i=1}^{N} \alpha_i \leq W \log \left(1 + \frac{\sum_{i \in S_k} P_i G_i}{N_0 W}\right) \ \forall \ k$

where $\mathcal{S} = \{S_1, \ldots, S_k\}$ is the set of all possible combinations of users.

The derivation of this region is shown, for example, in Chapter 14 of [9].

In addition to specifying feasible rates, our goal is to underline the desirable properties of a rate assignment. From a network perspective, the socially optimal rate assignment is the one which maximizes the total uplink utility of the users. Let function $U_i(\alpha_i)$ denote the utility that user i receives from uplink rate α_i. We define a socially optimal uplink rate allocation as follows.

Definition 2: A vector of uplink rates $\underline{\alpha}$ is a socially optimal uplink rate allocation if it is a solution to the following:

Uplink Rate Assignment Problem (URP)

$$\max_{\underline{\alpha} \in \Delta_U} \sum_{i=1}^{N} U_i(\alpha_i)$$

Although we have have specified the utility functions of users in very general terms, the form of these utility functions plays an important role. In this paper, we make the following assumptions about these utility functions.

Technical Assumption 1: Each user has a utility function of the form

$$U_i(\alpha_i) = \theta_i u(\alpha_i) \tag{1}$$

where $\theta_i \in [0, 1]$ is a privately held user-specific constant, and $u(\cdot)$ is a known and fixed function that is common to all users, and referred to as the homogenous component of the utility function. This homogeneous component of each user's utility satisfies the following properties:

1. $u(0) = 0$
2. $u'(\alpha_i) \geq 0$
3. $u''(\alpha_i) \leq 0$

Looking at (1), we see that each users's utility consists of two components: a diminishing returns component which is homogenous across all users, and a privately held user-specific constant, θ_i. We refer to this as user i's *private type*. This type can be generated in any number of ways, including as a random variable drawn from some probability distribution.

B. Downlink Rate Allocations

Just as we defined a feasible rate region for uplink rates, we must define a feasible rate region for downlink rates. Again, we work with an information-theoretic notion of capacity, this time using the standard feasible region associated with a Gaussian broadcast channel using time-division and fixed transmission power. The result is the following feasible rate region whose derivation is given, for example, in Chapter 14 of [9].

Definition 3: The *downlink feasible rate region* Δ_D is the set of rates $\underline{A}$ that satisfy the following conditions:

C1. $A_i \geq 0 \ \forall \ i \in \mathcal{N}$

C2. $\sum_{i=1}^{N} \frac{A_i}{W \log \left(1 + \frac{P_0 G_i}{N_0 W}\right)} \leq 1$

In the previous subsection, we formulated utility functions that give numerical values to how much a user values its allocated uplink rate. Similarly, we define utility functions that give numerical values to how much a user values is allocated downlink rate. Unlike the uplink utility, however, we do not assume that downlink rate exhibits diminishing returns; rather we assume that the utility of downlink rate is linear. Thus, the overall utility of user i can be expressed as:

$$V_i(\alpha_i, A_i) = \theta_i u(\alpha_i) + A_i \tag{2}$$

In economics literature, this type of utility function is known as quasi-linear. The uplink rate α_i is known as the *commodity of interest*, and the downlink rate is known as the *numeraire commodity*. In our work, the rational for this quasi-linear utility is that users have a significantly larger demand for downlink than for uplink (i.e. that the diminishing returns property of downlink rate occurs on a much larger scale than that of uplink rate). As we will see in the following sections, this quasi-linear structure is essential in developing our results.

It is important to note that since the goal is to use the downlink rate allocation in order to enforce a distributed and optimal uplink rate allocation, our only interest is in choosing a feasible downlink rate allocation. In fact, the mechanisms presented in the following sections will result in inefficiency on the downlink - an issue which will be addressed in later sections.

Finally, we also impose an admission control criteria on the number of users allowed in the system.

Technical Assumption 2: The network utilizes an admission control policy that limits the number of users in the system according to the following criteria:

$$N \leq \frac{W \log \left(1 + \frac{P_0 G_{min}}{N_0 W}\right)}{u\left(W \log \left(1 + \frac{P_i G_{max}}{N_0 W}\right)\right)}$$

where G_{min} is the minimum gain, and G_{max} is the maximum gain.

III. Joint Rate Allocation Mechanism

Any mechanism consists of three main stages. First, the *leader* (in this case the base station) announces the rules of the mechanism (known as an *outcome function*). Next, the *agents* (in this case the mobiles) submit information about their private type to the leader (known as *messages*). Finally, the leader allocates resources based on the information obtained from users in the second stage of the mechanism and the rules announced in the first stage of the mechanism. The goal of the mechanism is to ensure an allocation of the commodity of interest based on agents' private types, despite the fact that the true value of these types may be known only by the agents themselves.

Mathematically, mechanisms are described as a pair (M, g). Let M_i be the message space for user i, and $M = M_1 \times \ldots \times M_N$ be the cross-product of message spaces. In addition, let $g : M \to D \times R^{(N+1)}$ be an *outcome function* that maps messages to outcomes, where an outcome consists of a *decision rule* $D(\cdot)$ and *transfer functions* $t_i(\cdot),\ i = 1, \ldots, N$. The decision rule specifies the allocation of the commodity of interest (in our case the uplink rate), and the transfer functions specify the amount of numeraire commodity allocated to each user (in our case the downlink rate). For a more detailed discussion of mechanism design, see [3], [10], [11].

A. Specification of Joint Rate Allocation Mechanism

We now describe the *Joint Rate Allocation Mechanism* (JRAM).

1) *Message Space* - Each user selects a single value $w_i \in [0, 1]$ as its message. In other words, $M_i = [0, 1]\ \forall i$, and $M = [0, 1] \times \ldots \times [0, 1]$.

2) *Decision Rule* - The base station will assign uplink rates $\underline{\alpha}^*$ according to the following rule:

$$\underline{\alpha}^* = D(\underline{w}) = \arg \max_{\underline{\alpha} \in \Delta_U} \sum_{i=1}^{N} w_i u(\alpha_i) \tag{3}$$

where $u(\alpha_i)$ is the diminishing returns portion of the uplink utility, which is homogenous across all users and known at the base.

3) *Transfer Functions* - The base station will assign downlink rates $\underline{A}^*$ according to the following rule:

$$A_i^* = t_i(\underline{w}) = C_i + \sum_{j \neq i} w_j u(\alpha_j^*) - \sum_{j \neq i} w_j u(\alpha_j^i) \tag{4}$$

where $\underline{\alpha}^i$ is the solution to

$$\underline{\alpha}^i = \arg \max_{\underline{\alpha} \in \Delta_U} \sum_{j \neq i} w_j u(\alpha_j) \tag{5}$$

and $\underline{C}$ is the fixed proportional fair downlink rate allocation - i.e. is the solution to

$$\underline{C} = \arg \max_{\underline{C}' \in \Delta_D} \sum_{i=1}^{N} \log(C_i') \tag{6}$$

B. Analysis of the Mechanism

In most cases, the goal of mechanism design is simply to ensure a socially optimal allocation of the commodity of interest. However, since the numeraire commodity in the joint rate allocation mechanism is a physical quantity (i.e. downlink) there is an additional concern - namely, ensuring that the resulting downlink rate allocations are feasible. As such, we introduce the following theorem, for which the supporting lemmas can be found in Appendix I.

Theorem 1: Under Technical Assumption 1 and 2, the joint rate allocation mechanism (JRAM) implements Problem (URP)in dominant strategies, and results in a feasible downlink rate allocation.

Proof:

The first part of the proof is to show incentive compatibility of the mechanism. Recall that the total utility of user i is written as

$$V_i(\alpha_i, A_i) = \theta_i u(\alpha_i(\underline{w})) + A_i(\underline{w})$$

If the joint rate allocation mechanism does not implement Problem (URP)in dominant strategies, then there exists at least one user i and a value $\hat{w}_i \neq \theta_i$ such that

$$\theta_i u(\alpha_i(\hat{w}_i, w_{-i})) + A_i(\hat{w}_i, w_{-i}) > \theta_i u(\alpha_i(\underline{w})) + A_i(\underline{w}) \tag{7}$$

Let $\hat{\underline{\alpha}} = D(\hat{w}_i, w_{-i})$ be the solution to the maximization problem when user i bids $\hat{w}_i$. We can rewrite (7) as

$$\begin{aligned} \theta_i u(\hat{\alpha}_i) \quad &+ \quad C_i + \sum_{j \neq i} w_j u(\hat{\alpha}_j) - \sum_{j \neq i} w_j u(\alpha_j^i) \\ &> \quad \theta_i u(\alpha_i^*) + C_i + \sum_{j \neq i} w_j u(\alpha_j^*) - \sum_{j \neq i} w_j u(\alpha_j^i) \end{aligned}$$

which reduces to

$$\theta_i u(\hat{\alpha}_i) + \sum_{j \neq i} w_j u(\hat{\alpha}_j) > \theta_i u(\alpha_i^*) + \sum_{j \neq i} w_j u(\alpha_j^*) \tag{8}$$

However, since by definition

$$\underline{\alpha}^* = \arg \max_{\underline{\alpha} \in \Delta_U} \sum_{j=1}^{N} w_j u(\alpha_j) = \arg \max_{\underline{\alpha} \in \Delta_U} \sum_{j=1}^{N} \theta_j u(\alpha_j) \tag{9}$$

then (8) is clearly a contradiction. Hence, the joint rate allocation mechanism implements (URP)in dominant strategies. That this mechanism results in a socially optimal and feasible uplink rate allocation follows directly from (9). Note that since the joint rate allocation mechanism is an extension of a VCG mechanism, the preceding proof is an extension of the known proof of incentive compatibility for VCG mechanisms (see e.g. [3]).

The feasibility of downlink rates requires several lemmas, and is shown in Appendix I.

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IV. Conclusions

In this paper, we have constructed a joint rate allocation mechanism that results in feasible uplink and downlink rate assignments. Furthermore, we have shown that by leveraging users' demand for downlink, the mechanism implements the socially optimal rate assignment problem in dominant strategies, even when users act strategically.

Although this paper presents an interesting first result, there is room for future work and improvement. In constructing the joint rate allocation mechanism, we assume information theoretic AWGN MAC and broadcast channels for the uplink and downlink, respectively. These results can, in fact, be extended to a large class of wireless channels so long as certain properties (convexity and coordinate convexity) are maintained.

Similarly, we examine a very specific utility function of the form $U_i(\alpha_i) = \theta_i u(\alpha_i)$. The results can be extended to a large class of utility functions, so long as they satisfy the same three conditions we require of the homogenous component $u(\alpha_i)$. However, when considering utility functions that do not have a homogenous component known to the base, the users need to report an entire function rather than a single value. It would be interesting to examine this work in the context of *scalar VCG mechanisms*, which implement problems in Nash equilibrium (rather than dominant strategies) but only require users to report a single parameter value (see [12], [13]).

Finally, it should be noted that we have not addressed the efficiency of the downlink rate allocations, since we are interested in the downlink only as a tool for regulating uplink rate assignments. However, since the downlink rate allocations given by (4) are inherently inefficient from the perspective of downlink allocations, further study is needed - particularly with regard to the impact of channel conditions, the number of users, and the admission control criteria.

APPENDIX I
PROOF OF THEOREM 1

Lemma 1: Consider Δ_D as given in Definition 3. The proportional fair downlink allocation $\underline{\hat{C}}$ is given by:

$$\hat{C}_i = \frac{W \log\left(1 + \frac{P_0 G_i}{N_0 W}\right)}{N} \;\forall\; i \in \mathcal{N} \tag{10}$$

Proof: Recall the proportional fair downlink allocation given by (6). Since we are maximizing a strictly concave function over a linear region, there is a unique solution which can be found by solving the dual problem [14]. As such, we introduce the LaGrange function:

$$\begin{aligned}\mathcal{L}(\underline{\hat{C}}, \lambda) &= \sum_{i=1}^{N} \log(\hat{C}_i) - \lambda\left(\sum_{i=1}^{N} \frac{\hat{C}_i}{W\log\left(1+\frac{P_0 G_i}{N_0 W}\right)} - 1\right) \\ &= \sum_{i=1}^{N}\left(\log(\hat{C}_i) - \frac{\lambda \hat{C}_i}{W\log\left(1+\frac{P_0 G_i}{N_0 W}\right)}\right) + \lambda\end{aligned}$$

We can now formulate the dual problem as:

$$\begin{aligned}\min_{\lambda} \quad \max\nolimits_{\underline{\hat{C}}} \quad & \mathcal{L}(\underline{\hat{C}}, \lambda) \\ = \quad & \min_{\lambda}\max_{\underline{\hat{C}}} \sum_{i=1}^{N}\left(\log(\hat{C}_i) - \frac{\lambda \hat{C}_i}{W\log\left(1+\frac{P_0 G_i}{N_0 W}\right)}\right) + \lambda\end{aligned}$$

Using first-order conditions to solve, we get

$$\lambda^* = \min_{\lambda} \sum_{i=1}^{N}\left(\log\left(\frac{W\log\left(1+\frac{P_0 G_i}{N_0 W}\right)}{\lambda}\right) - 1\right) + \lambda = N$$

and

$$\hat{C}_i = \frac{W\log\left(1+\frac{P_0 G_i}{N_0 W}\right)}{\lambda} = \frac{W\log\left(1+\frac{P_0 G_i}{N_0 W}\right)}{N}$$

■

Lemma 2: Under Technical Assumption 1 and 2, the downlink rate allocation described by (4) is a feasible rate allocation.

Proof: First, consider the lower bound $A_i \geq 0$ from Condition C1 in Definition 3. We note the following:

$$\begin{aligned}\sum_{j\neq i} w_j u(\alpha_j^i) \quad - \quad & \sum_{j\neq i} w_j u(\alpha_j^*) \\ = \quad & \sum_{j\neq i} w_j u(\alpha_j^i) + w_i u(0) - \sum_{j\neq i} w_j u(\alpha_j^*) \\ \leq \quad & \sum_{j=1}^{N} w_j u(\alpha_j^*) - \sum_{j\neq i} w_j u(\alpha_j^*) \\ = \quad & w_i u(\alpha_i^*)\end{aligned}$$

where the first equality comes from Technical Assumption 1, and the inequality comes from the definition of $\underline{\alpha}^*$. We also note that

$$w_i u(\alpha_i^*) \leq u(\alpha_i^*) \leq u\left(W\log\left(1+\frac{P_i G_i}{N_0 W}\right)\right)$$

where the first inequality comes from Technical Assumption 1, and the second inequality comes from the definition of Δ_U. We now have:

$$\begin{aligned}A_i^* \quad &= \quad C_i + \sum_{j\neq i} w_j u(\alpha_j^*) - \sum_{j\neq i} w_j u(\alpha_j^i) \\ &\geq \quad C_i - u(1 + W\log(1+\frac{P_i G_i}{N_0 W})) \\ &\geq \quad \frac{W\log(1+\frac{P_0 G_i}{N_0 W})}{N} - u(1+W\log(1+\frac{P_i G_i}{N_0 W})) \\ &\geq \quad 0\end{aligned}$$

where the second inequality comes from Lemma 1, and the final inequality comes from Technical Assumption 2.

Next, we consider the upper bound $\sum_{i=1}^{N} \frac{A_i}{\log(1+\frac{P_0 G_i}{N_0 W})} \leq 1$ from Condition C2 in Definition 3. Note that by definition, $\sum_{j\neq i} w_j u(\alpha_j^i) \geq \sum_{j\neq i} w_j u(\alpha_j^*)$. This gives

$$A_i^* = C_i + \sum_{j\neq i} w_j u(\alpha_j^*) - \sum_{j\neq i} w_j u(\alpha_j^i) \leq C_i \;\forall\; i$$

Since $\underline{C}$ is feasible and Δ_D is a coordinate convex region, $\underline{A}^*$ must also be feasible.

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