Leveraging Downlink for Regulation of Distributed Uplink CDMA

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***Abstract*— In this paper, we formulate an optimal rate allocation scheme for uplink and downlink in a single-cell CDMA system. Using concepts from game theory, we show that it is possible to construct a game in which the socially optimal rate allocation arises as a subgame perfect Nash equilibrium. We further show that this equilibrium can be achieved via a distributed algorithm at the uplink using practical signaling mechanisms.**

I. Introduction

Auctioning schemes have proved to be extremely powerful tools when used to address adaptive and distributed allocation of resources in a network with constraints. In particular, the economic notion of pricing can be used to design appropriate signaling mechanisms whose goal is to align the behavior of individual users with the global well being (see e.g. [1], [2], [3], [4], [5], [6]). Unlike prices in an economic system, however, these signals do not represent a true cost to the users; hence, they offer little guarantee that the users will exhibit the desired behavior in the face of a potential tragedy of commons situation [7].

In many works, it is common to address this problem using dollar-valued pricing schemes in which users pay for service (see e.g. [8], [9], [10], [11], [12]). Such an approach requires the implementation of complicated pricing schemes by the service provider, however. The goal of this paper is to develop distributed rate assignment algorithms for the uplink of a single-cell CDMA network that do not rely on dollar valued pricing schemes. Specifically, we are interested in incorporating a centralized downlink scheduler to provide efficient rate allocations while mitigating the tragedy of commons phenomenon over the uplink. To the best of our knowledge, this is a novel approach which has not previously been studied.

In wireless communications literature, the tragedy of commons problem has typically been addressed by modeling an explicit energy cost to the users. For example, the authors in [13] consider power consumption as a direct cost to the users. A Nash equilibrium solution is obtained when users have an interest in maximizing their own utility, defined as the ratio of rate to power. This approach, however, results in a sub-optimal resource allocation. This work can be extended to capture the affect of receiver design, as in [14] where the authors show that receiver design can be used to induce a more efficient Nash equilibrium. It is our goal to show that even when using the traditional matched filter receiver, it is possible to obtain an efficient Nash equilibrium. In order to do so, we draw on concepts from economics by leveraging the downlink.

In economics literature, an elegant way to address the tragedy of commons problem is the introduction of a *numeraire commodity* - a commodity for which users have extremely large or infinite demand. Rather than being charged for the original commodity of interest, users are "paid" in the numeraire commodity for consuming the original commodity in a socially responsible way [15]. In this paper, we use the inherent asymmetry between uplink and downlink and (in terms of control and demand) to model the downlink bandwidth as a numeraire commodity. In other words, we use a centralized downlink scheduler to enforce distributed uplink scheduling. In particular, we show that with an appropriately designed downlink scheduler the socially optimal uplink rate allocation emerges as a subgame perfect Nash equilibrium. We also provide practical distributed algorithms to arrive at such an equilibrium.

The rest of this paper is organized as follows. Section II describes the uplink and downlink rate assignment problems. Section III describes the formulation of a non-cooperative rate assignment game, in which the base station assigns downlink rates to enforce the distributed uplink rate assignment. In this section we show that the optimal uplink rate assignment emerges as a subgame perfect Nash equilibrium of the game. Section IV describes a practical, distributed signaling mechanism which can be used to achieve the Nash equilibrium of the game. Finally, Section V presents our conclusions and areas for future work.

II. Socially Optimal Rate Allocations

In order to describe the rate assignment problem for a single-cell CDMA network, we use the following notation. There are N mobiles with chip bandwidth W. Each mobile transmits over the air to the base station with rate α_i. The base station transmits over the air to each mobile with rate A_i.

A. Uplink Rate Allocations

In order to define the notion of a socially optimal uplink rate allocation, we must first define the notion of a feasible rate region. In any wireless system, the definition of what

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constitutes a feasible rate-power pair is highly dependent upon both the application and design of a particular system. Constraints may include minimum or maximum power constraints, interference limits, minimum rate guarantees, QoS metrics, or delay requirements. Since the focus of this work is a data-optimized (DO) network, we focus on a commonly used definition of uplink feasible rates which depends on both a target $\frac{E_b}{N_t}$ (denoted by γ) and a target interference level (denoted by K). A more detailed explanation of these feasibility criteria can be found in the 3GPP2 standards for CDMA2000 [16].

An important (and often neglected) issue in high data rate CDMA networks is the performance degradation due to multipath interference when low spreading gains are used [17], [18]. While the standard Gaussian approximation used for performance analysis of a matched filter receiver is valid for high spreading gain, it becomes less and less valid as the spreading gain decreases. As such, we restrict our attention to uplink transmission rates that satisfy $\alpha_i \leq \frac{W}{4}$. This gives a spreading gain which the authors in [17] have shown to exhibit only moderate performance degradation due to multipath interference.

With these issues in mind, we introduce our uplink feasible rate region whose derivation is shown in Appendix I.

Definition 1: The *uplink feasible rate region* Δ_U is the set of uplink rates $\underline{\alpha}$ such that

$$\Delta_U = \{\underline{\alpha} : \sum_{i=1}^{N} \frac{\alpha_i}{\alpha_i \gamma + W} \leq \frac{K}{\gamma(1+K)},\ 0 \leq \alpha_i \leq \frac{W}{4}\ \forall\ i\}$$

where γ and K are pre-defined constants.

Having defined what constitutes a feasible rate, our goal is to underline the desirable properties of a rate assignment. From a network perspective, the socially optimal rate assignment is the one which maximizes the total utility of the users. Let function $U_i(\alpha_i)$ denote the utility that user i receives from uplink rate α_i. We define a socially optimal uplink rate allocation as follows.

Definition 2: A vector of rates $\underline{\alpha}$ is a *socially optimal uplink rate allocation* if it is a solution to the following:

Problem P

$$\max_{\underline{\alpha} \in \Delta_U} \sum_{i=1}^{N} U_i(\alpha_i)$$

It is standard in optimization literature to assume that the utilities of the mobiles are strictly increasing and twice differentiable functions of α_i. Here, we introduce an additional technical requirement on these functions.

Technical Assumption 1: The mobiles' utility functions $U_i(\cdot)$ are assumed to be *critically concave*, i.e. they satisfy the condition $U''(x) \leq \frac{-1}{x^2}$.

Notice that this not only ensures that uplink utility satisfies the property of diminishing returns, but that the utility function is, in some sense, "sufficiently concave." We now introduce the following theorem regarding socially optimal uplink rate assignments, whose proof is given in Appendix II.

Theorem 1: Under Technical Assumption 1, there exists a value $\mu^* \geq 0$ for which $\underline{\alpha}^* = (\alpha_1^*, \ldots, \alpha_N^*)$,

$$\alpha_i^* = \arg \max_{0 \leq \alpha \leq \frac{W}{4}} \left(U_i(\alpha_i) - \mu^* \frac{\alpha_i}{\alpha_i \gamma + W} \right) \quad (1)$$

is the socially optimal solution to Problem P.

B. Downlink Rate Allocations

On the uplink, we derived the feasible rate region using target values for $\frac{E_b}{N_t}$ and total interference. On the downlink, we simply work with the standard Gaussian broadcast channel using time-division and fixed transmission power. The result is the following feasible rate region, whose derivation is shown, for example, in [19].

Definition 3: The *downlink feasible rate region* Δ_D is the set of downlink rates $\underline{A}$ such that

$$\Delta_D = \{\underline{A} : \sum_{i=1}^{N} \eta_i A_i \leq \eta_0\}$$

where $\underline{\eta}$ are constants which depend on transmission power and the channel gains.

Since the goal is to use the downlink rate allocation in order to enforce a distributed and optimal uplink rate allocation, our only interest is in choosing a downlink rate allocation that exists on the boundary of the feasibility region Δ_D. Therefore, we define an efficient downlink rate allocation as follows.

Definition 4: A vector of downlink rates $\underline{A}$ is *efficient* if it satisfies the condition $\sum_{i=1}^{N} \eta_i A_i = \eta_0$.

Note that while we assume the mobiles' utility for uplink rate satisfies the diminishing returns property, we assume the mobiles' utility for downlink rate is linear. The rational for this is that users have a significantly larger demand for downlink bandwidth than for uplink. Thus, the mobiles have quasi-linear utility functions of the form $u_i = U_i(\alpha_i) + A_i$. As we will see, this quasi-linear structure is essential in developing our results.

III. Non-Cooperative Rate Assignment Game

In order to examine the behavior of the mobiles and base station, we formulate the optimal rate assignment problem as a multi-stage game with two types of players: a base station and the mobiles. In this scenario, the base station is a benevolent player whose goal is to ensure a socially optimal and efficient rate allocation. The base station's "control knob" is its ability to schedule the downlink rates at a later stage. The mobiles are selfish players interested in choosing their uplink rates so as to maximize their own utility without consideration for the social welfare.

The game is structured such that the mobiles know apriori that their downlink rates A_i will be assigned according to a rule of the form

$$A_i(\mu, \alpha_i, \mathcal{C}) = \mathcal{C} - \mu \frac{\alpha_i}{\alpha_i \gamma + W} \quad (2)$$

where μ is a parameter chosen by the base station in the first stage of the game, α_i is the rate chosen by mobile i in the second stage of the game, and $\mathcal{C}$ is the maximum possible

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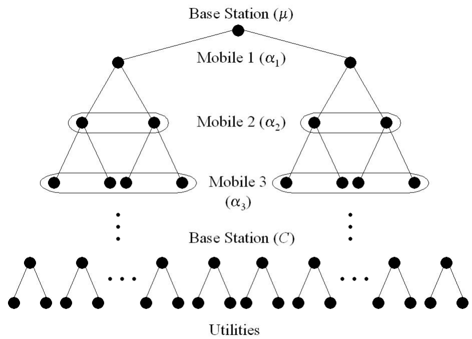

Fig. 1. Extensive-Form Representation of the Rate Assignment Game

downlink rate chosen by the base station in the last stage of the game.

Intuitively, the game consists of the following three stages:

1. *Parameter Announcement* - Taking into account the utilities of the mobiles, the base station announces its choice of parameter μ, which is observed by all the mobiles.
2. *Uplink Rate Selection* - Each mobile selects its uplink rate, α_i, in an attempt to maximize its own utility. The choice of α_i is observed by the base station but not by any of the other mobiles.
3. *Downlink Rate Selection* - The base station computes $\mathcal{C}$ and assigns downlink rates according to (2).

Figure 1 shows the extensive form representation for this game. (Note that for illustrative purposes, the quantities μ, $\underline{\alpha}$, and $\mathcal{C}$ are shown as being discrete-valued.) The base station acts first, choosing the value of μ and broadcasting that information to the mobiles. Next, the mobiles use information from the base station to simultaneously select their uplink rates $(\alpha_1, \ldots, \alpha_N)$. After all of the mobiles have selected a uplink rate, the base station selects the value of $\mathcal{C}$ and assigns the downlink rates $\underline{A}$.

Note that in an extensive form representation of a game, the concept of *information sets* is commonly used to model simultaneous moves. An information set is a collection of nodes that represent what a player knows about how the game has been played so far. In other words, a player is unable to tell which of the decision nodes in an information set it is actually at [15], [20]. The ovals in Figure 1 represent the information sets of this game. So, although Mobile 2 knows whether he is playing in the left or right branch of the original game, he does not know whether he is playing at the left or right branch following Mobile 1's choice of rate. (Since none of the mobiles are able to observe the actions of other mobiles, the ordering of the mobiles plays no role in the outcome of the game.) The information sets at the last stage of the game are singletons, since the base station is able to observe the uplink rate selections of all of the mobiles.

At this point, we make two assumptions which are inherent to the game.

Technical Assumption 2: The base station has perfect information regarding the utility of each mobile - that is, it knows $U_i(\alpha_i)$ for each mobile.

Technical Assumption 3: Although mobiles know the rule $A_i = \mathcal{C} - \mu\frac{\alpha_i}{\alpha_i\gamma+W}$ according to which their downlink rates will be allocated, they do *not* know how the parameters $\mathcal{C}$ and μ are generated.

Both of these assumptions are of critical importance - an issue which is discussed further in Section V.

A. Game Formulation

Formally, we denote by $F = [\mathcal{N}, \{\Sigma_i\}, \{u_i(\cdot)\}]$ the non-cooperative rate assignment game. Let $\mathcal{N} = \{0, 1, \ldots, N\}$ denote the set of players, where player $i = 0$ is the base station and players $i = 1, \ldots, N$ are the mobiles. The strategy space for the base station is $\Sigma_0 = \{(\mu, \mathcal{C}) : \mu \geq 0, \mathcal{C} \geq 0\}$, and for the mobiles is $\Sigma_i = \{\alpha_i : 0 \leq \alpha_i \leq \frac{W}{4}\}$. Since the base station is a benevolent player whose only goal is to ensure socially optimal and efficient rate allocations, the utility of the base station is defined as:

$$
\begin{aligned}
u_0(\mu, \underline{\alpha}, \mathcal{C}) &= \left[\sum_{i=1}^{N} U_i(\alpha_i)\right]\mathcal{I}(\underline{\alpha}) \\
&\quad + \left[\sum_{i=1}^{N} \eta_i A_i(\mu, \alpha_i, \mathcal{C})\right]\mathcal{I}(\underline{A}(\mu, \underline{\alpha}, \mathcal{C}))
\end{aligned}
$$

where

$$
\mathcal{I}(\underline{\alpha}) = \begin{cases} 1 & \text{if } \underline{\alpha} \in \Delta_U \\ 0 & \text{else} \end{cases}
$$

$$
\mathcal{I}(\underline{A}) = \begin{cases} 1 & \text{if } \underline{A} \in \Delta_D \\ 0 & \text{else} \end{cases}
$$

The mobiles are assumed to have quasi-linear utility functions of the form

$$
u_i(\mu, \alpha_i, \mathcal{C}) = U_i(\alpha_i) + A_i(\mu, \alpha_i, \mathcal{C})
$$

where $U_i(\cdot)$ is the mobiles' uplink utility function, and the downlink rate is the numeraire commodity for which utility is linear [15].

B. Socially Optimal Rate Allocation and Nash Equilibrium

Having constructed our rate assignment game, a natural question is whether or not the socially optimal rate allocation will emerge as an outcome of this game. Recall the definition of a Nash equilibrium ([15], [20]):

Definition 5: A strategy profile σ is a Nash equilibrium if and only if

$$
u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma'_i, \sigma_{-i}) \quad \forall \quad \sigma'_i \in \Sigma_i, \quad \sigma'_i \neq \sigma_i
$$

for every player i.

It is our goal in this section to show that the socially optimal rate allocation arises from the Nash equilibrium for our game. We have already shown that there exists a value μ^* such that the rates $\underline{\alpha}^*$ are the solution to Problem P when each α_i^* is assigned according to (1). Furthermore, we notice that the rates $\underline{A}^*$ are an efficient downlink rate allocation when each A_i^* is assigned according to $A_i^* = \mathcal{C}^* - \mu^*\frac{\alpha_i^*}{\alpha_i^*\gamma+W}$ and $\mathcal{C}^*$ is assigned according to:

$$
\mathcal{C}^* = \frac{\eta_0 + \mu^* \sum_{i=1}^{N}\left(\eta_i \frac{\alpha_i^*}{\alpha_i^*\gamma+W}\right)}{\sum_{i=1}^{N} \eta_i}
$$

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The efficiency of the rates $\underline{A}^*$ can be easily seen by verifying that $\sum_{i=1}^{N} \eta_i A_i^* = \eta_0$.

We now show that the efficient and socially optimal rates $(\underline{A}^*, \underline{\alpha}^*)$ is, indeed, a Nash equilibrium for Game F.

Theorem 2: Let $\sigma = \{(\mu^*, \mathcal{C}^*), \alpha_1^*, \ldots, \alpha_N^*\}$. Then the strategy profile σ is a Nash equilibrium for game F.

Proof:

To show that σ is a Nash equilibrium for game F, we must first show that $\forall\ i,\ u_i(\mu^*, \alpha_i^*, \mathcal{C}^*) \geq u_i(\mu^*, \alpha_i, \mathcal{C}^*) \quad \forall \quad \alpha_i \neq \alpha_i^*$. Written another way, we say that α_i^* must satisfy

$$\begin{aligned}\alpha_i^* &= \arg \max_{0 \leq \alpha \leq \frac{W}{4}} \left(U_i(\alpha) + \mathcal{C}^* - \mu^* \frac{\alpha}{\gamma\alpha + W} \right) \\ &= \arg \max_{0 \leq \alpha \leq \frac{W}{4}} \left(U_i(\alpha) - \mu^* \frac{\alpha}{\gamma\alpha + W} \right)\end{aligned}$$

This is exactly the definition of α_i^*.

Now, recall that the base station has utility

$$\begin{aligned}u_0(\mu, \underline{\alpha}, \mathcal{C}) &= \left[\sum_{i=1}^{N} U_i(\alpha_i) \right] \mathcal{I}(\underline{\alpha}) \\ &+ \left[\sum_{i=1}^{N} \eta_i A_i(\mu, \alpha_i, \mathcal{C}) \right] \mathcal{I}(\underline{A}(\mu, \underline{\alpha}, \mathcal{C}))\end{aligned}$$

By construction, it is clear that the utility of the base is maximized by μ^* and $\mathcal{C}^*$ when the mobiles choose rates $\underline{\alpha}^*$, and we are done. ■

The implication of Theorem 2 is significant. We have shown that if the base station acts appropriately, the socially optimal solution emerges as a Nash equilibrium even when mobiles choose their downlink rates simultaneously and without regard to their impact on the system as a whole!

C. Subgame Perfect Nash Equilibrium

While the idea of Nash equilibrium is commonly used in game theory, this concept can fall short in multi-stage games such as the one formulated above. In particular, multi-stage games may give rise to Nash equilibria that would be considered irrational from the perspective of a user who has observed certain past actions [15], [20]. Take, for example, our strategy σ from the previous section. We know that it is in the base station's best interest to choose $\mu = \mu^*$, hence it is the mobiles' best interest to choose $\alpha_i = \alpha_i^*$. Assume, however, that for some reason the base actually chooses $\mu \neq \mu^*$. Our strategy σ implies that the mobiles will still choose $\alpha_i = \alpha_i^*$, even though this may not maximize their utility (since the base station did not play as expected). In order to remedy this, we use the idea of subgames and subgame perfect Nash equilibrium ([15], [20]).

Definition 6: A subgame of game F is a subset of game F such that it begins with a single node, contains all successors of that node, and contains all nodes in any given information set if it contains at least one node from that information set.

Definition 7: A strategy σ is a *subgame perfect Nash equilibrium* for game F if it induces a Nash equilibrium on every subgame of F.

In other words, a subgame perfect Nash equilibrium must be consistent with maximizing users' utilities, even when previous moves were "unexpectedly irrational."

In the previous section, we studied a Nash equilibrium strategy profile σ which consisted of specific values. When examining subgame perfect Nash equilibrium strategies, however, we need to construct strategy profiles which consist of *functions*, rather than specific values. These functions articulate each user's action as a response to previous players' observed actions. Therefore, we define the following functions:

$$\alpha_i = h_i^*(\mu) = \arg \max_{0 \leq \alpha \leq \frac{W}{4}} \left(U_i(\alpha) - \mu \frac{\alpha}{\alpha\gamma + W} \right)$$

$$\mathcal{C} = f^*(\mu, \underline{\alpha}) = \frac{\eta_0 + \mu \sum_{i=1}^{N} \left(\eta_i \frac{\alpha_i}{\alpha_i\gamma + W} \right)}{\sum_{i=1}^{N} \eta_i} \tag{3}$$

Now, we relate these equations to the idea of a subgame perfect Nash equilibrium.

Theorem 3: Let $\sigma' = \{(\mu^*, f^*(\cdot)), h_1^*(\cdot), \ldots, h_N^*(\cdot)\}$. Then σ' is a subgame perfect Nash equilibrium for game F.

Proof:

We begin by categorizing the subgames of F into three groups:

1. the subgames which originate with the base station's choice of maximum downlink rate (final stage)
2. the subgames which originate with the first mobile's choice of uplink rate (second stage)
3. the game itself

To show that σ' induces a Nash equilibrium for the first group of subgames, we simply note that

$$\sum_{i=1}^{N} \eta_i A_i = \sum_{i=1}^{N} \eta_i (f_i(\mu, \underline{\alpha}) - \mu \frac{\alpha_i}{\gamma\alpha_i W}) = \eta_0$$

Thus $\mathcal{C} = f_i(\mu, \underline{\alpha})$ maximizes the base station's utility for any given choice of μ and $\underline{\alpha}$.

To show that σ' induces a Nash equilibrium for the second group of subgames, recall that we assume mobiles do not know how the base station chooses the value $\mathcal{C}$. In other words, the mobiles see the base station's strategy as $\mathcal{C}$, not $f(\mu, \underline{\alpha})$. Thus, in order to show that σ' induces a Nash equilibrium for the second group of subgames we must show that for every i and for any μ, $u_i(\mu, h_i(\mu), \mathcal{C}) \geq u_i(\mu, \alpha, \mathcal{C}) \quad \forall \quad \alpha \neq h_i(\mu)$. Written another way, we can say that $h_i(\mu)$ must satisfy

$$\begin{aligned}h_i(\mu) &= \arg \max_{0 \leq \alpha \leq \frac{W}{4}} \left(U_i(\alpha) + \mathcal{C} - \mu \frac{\alpha}{\gamma\alpha + W} \right) \\ &= \arg \max_{0 \leq \alpha \leq \frac{W}{4}} \left(U_i(\alpha) - \mu \frac{\alpha}{\gamma\alpha + W} \right)\end{aligned}$$

This is exactly the definition of $h_i(\mu)$.

Finally, to show that σ' induces a Nash equilibrium for the game itself we only need observe that $h_i(\mu^*) = \alpha_i^*$ and $f(\mu^*, \underline{\alpha}^*) = \mathcal{C}^*$. Combining this with the result from Theorem 2 gives the desired result. ■

Notice that the proof of Theorem 3 shows that when all players are rational, the strategy σ' coincides with σ, which

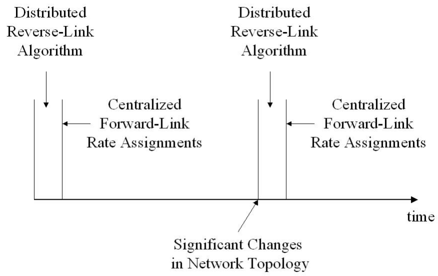

Fig. 2. Relative Time-Scales for Distributed Implementation of the Non-Cooperative Rate Assignment Game

results in an efficient and socially optimal rate allocation. In the unlikely event that $\mu \neq \mu^*$, the mobiles will still choose their uplink rates consistent with maximizing their own utilities. Similarly, if $\alpha_i \neq \alpha_i^*$ for any subset of mobiles i, the base will still assign downlink rates in an efficient manner.

IV. Distributed Computation of the Nash Equilibrium

While Theorem 3 provides an extremely useful result regarding the existence of a socially optimal subgame perfect Nash equilibrium, this is only part of the story. Although (1) shows how mobiles can calculate their optimal rates α_i^* in a distributed manner given the value μ^*, we are also interested in how to find μ^* in a distributed manner. Although it is possible for the base station to calculate this quantity, it is obviously undesirable - particularly in scenarios where mobiles may enter or leave the system, requiring a new round of play.

In order to develop a practical implementation of the game described above we consider a two-time scale system, as shown in Figure 2. The base station and mobiles run a distributed algorithm for finding μ^* and $\underline{\alpha}^*$ on a fast time-scale. Once this algorithm converges, the base station will assign downlink rates according to (2) and (3). As significant changes in topology occur (on a slow time-scale), the process is repeated.

We now introduce the following distributed algorithms for arriving at μ^*, and $\underline{\alpha}^*$.

Base Algorithm

Each base station announces its strategy μ, which evolves according to the following difference equation:

$$\Delta\mu = \begin{cases} \beta(\sum_{i=1}^{N} \frac{\alpha_i}{W+\gamma\alpha_i} - \frac{K}{\gamma(1+K)}) & \text{if } \mu > 0 \\ \beta[\sum_{i=1}^{N} \frac{\alpha_i}{W+\gamma\alpha_i} - \frac{K}{\gamma(1+K)}]^+ & \text{if } \mu = 0 \end{cases} \quad (4)$$

where β is a constant.

Mobile Algorithm

Each mobile reacts to the regulatory signal from the base station by adjusting its uplink rate such that

$$\alpha_i = \arg \max_{0 \leq \alpha \leq \frac{W}{4}} \left(U_i(\alpha) - \frac{\alpha}{W+\gamma\alpha}\mu \right) \quad (5)$$

It has been shown in previous work [5], [6] that the distributed algorithm described by (4)-(5) converges to the solution to Problem P.

V. Conclusions

We have shown how, using the economic idea of quasi-linear utility, it is possible to model downlink bandwidth in a CDMA network as a numeraire commodity. Thus, an appropriately designed downlink scheduler can be used to enforce a distributed and optimal uplink rate allocation. Furthermore, we show that this optimal rate allocation emerges as a Nash equilibrium of a multi-stage game which can be implemented using practical signaling mechanisms. Although the scheme provided in this paper addresses a specific uplink-downlink pair, the idea of using a centralized downlink scheduler to enforce a distributed uplink allocation can be used to replace complicated dollar-valued pricing structures in a wide range of situations where the tragedy of commons phenomenon presents itself.

Although this paper presents an interesting first result, there is room for further study. One of the assumptions made in this paper is that mobiles do not know how the values $\mathcal{C}$ and μ are computed (i.e. mobiles are price-taking users). It is possible, however, for mobiles to attempt to learn how these values are calculated. In the case of congestion control, [1], [21] and [22] have shown that when users understand the role of prices and their own impact on the system, the efficiency of the system decreases. An important area of future work will be to investigate the impact of non-price-taking users on the system. In addition, the game presented in this paper assumes the base station has complete knowledge about the users' utilities. A very interesting avenue of future work is to design an incentive-compatible mechanism which guarantees that rational users will disclose their utility functions in a truthful manner. Finally, the formulation in this paper assumes static channel conditions for the duration of the game. Incorporating the fading characteristics of the channel, assumed to be an inherent randomness in the game (e.g. see [23]), will be another important area of future work.

Appendix I

Let p_i be the power with which mobile i transmits to the base station. We say a vector of uplink rates $\underline{\alpha}$ is a feasible solution if there exists a vector of transmission powers $(p_1, \ldots, p_N)$ that satisfies the following conditions:

1. $\frac{\frac{W}{\alpha_i} p_i g_i}{N_0 W + \sum_{j \neq i} p_j g_j} = \gamma \ \forall i$
2. $\sum_{i=1}^{N} p_i g_i \leq K N_0 W$

where N_0 is the thermal noise level, and γ and K are predefined constants.

Rearranging Condition 1 and summing over all users gives:

$$\sum_{i=1}^{N} p_i g_i = \sum_{i=1}^{N} \frac{\gamma(N_0 W + \sum_{j=1}^{N} p_j g_j)}{\frac{W}{\alpha_i} + \gamma}$$

Substituting $r_i = \frac{1}{\frac{W}{\alpha_i}+\gamma}$ and $z = \sum_{j=1}^{N} p_j g_j$ yields

$$z = \gamma(N_0 W + z) \sum_{i=1}^{N} r_i \quad (6)$$

Notice that, using the above substitution, Condition 2 is nothing but $z \leq K N_0 W$. Thus, by rearranging (6) and substituting into Condition 2, we get the following:

$$\sum_{i=1}^{N} r_i \leq \frac{K}{\gamma(1+K)}$$

Substituting α_i back in gives the feasibility region Δ_U.

APPENDIX II

Notice that Problem P, as it is written, is not a convex optimization problem - hence may not be solvable using dual theory. In order to show that Problem P can be transformed into a convex optimization problem, we introduce the change of variable $r_i = \frac{\alpha_i}{\alpha_i \gamma + W}$. This gives the following problem:

Problem P1

$$\max_{\underline{r}} \sum_{i=1}^{N} U_i\left(\frac{r_i W}{1-\gamma r_i}\right)$$

s.t.

$$0 \leq r_i \leq \frac{\frac{W}{4}}{\frac{W}{4}\gamma + W}$$

$$\sum_{i=1}^{N} r_i \leq \frac{K}{\gamma(1+K)}$$

Now we show that Problem P1 is a concave optimization problem in terms of primal variables r_i:

$$\begin{aligned} & \frac{\partial^2}{\partial r_i^2}\left(U_i\left(\frac{r_i W}{1-\gamma r_i}\right)\right) \\ = \; & U_i''\left(\frac{r_i W}{1-\gamma r_i}\right)\frac{W^2}{(1-\gamma r_i)^4} + U_i'\left(\frac{r_i W}{1-\gamma r_i}\right)\frac{2\gamma W}{(1-\gamma r_i)^3} \\ \leq \; & \frac{-(1-\gamma r_i)^2}{r_i^2 W^2}\Big)\frac{W^2}{(1-\gamma r_i)^4} + \frac{(1-\gamma r_i)}{r_i W}\Big)\frac{2\gamma W}{(1-\gamma r_i)^3} \\ = \; & \frac{2\gamma r_i - 1}{r_i^2(1-\gamma r_i)^2} \leq 0 \end{aligned}$$

where the first inequality results from Technical Assumption 1, and the last inequality comes from $r_i \leq \frac{\frac{W}{4}}{\frac{W}{4}\gamma + W} = \frac{1}{\gamma+4}$ and the choice of $\gamma \leq 6$ dB [24].

Since this problem maximizes a strictly concave function (over the range of interest) subject to a linear constraint, there is no duality gap ([25], Proposition 5.3.1, page 512) and we can address this using the dual methods.

Consider the dual function associated with Problem P1:

$$\mathcal{G}(\mu) = \max_{0 \leq \underline{r} \leq \frac{1}{\gamma+4}} \sum_{i=1}^{N} \left(U_i\left(\frac{r_i W}{1-\gamma r_i}\right) - \mu r_i \right) + \mu \frac{K}{\gamma(1+K)}$$

Now, let $\mu^* = \min_\mu \mathcal{G}(\mu)$ and

$$r_i^* = \arg \max_{0 \leq r \leq \frac{1}{\gamma+4}} \left(U_i\left(\frac{rW}{1-\gamma r}\right) - \mu^* r \right)$$

By construction, μ^* is the Lagrange multiplier for Problem P1, and $\underline{r}^*$ is the solution to Problem P1. Transforming back to the original variables $\underline{\alpha}$ gives the desired result.

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