Leveraging Downlink for Efficient Uplink Allocation in a Single-Hop Wireless Network

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***Abstract*— In this paper, we consider joint uplink and downlink rate allocations over a multi-access/broadcast channel pair in a single-cell system. We assume a network of heterogeneous users, where the uplink utility of each user is held as private information and is unknown to the base station. The challenge is to design an optimal rate allocation scheme in the presence of such incomplete information when considering strategic users. Here, we provide an incentive-compatible mechanism that leverages downlink demand to ensure that users truthfully reveal their uplink utilities, enabling the socially optimal uplink rate allocation. In addition, we give numerical results on a kind of efficiency of downlink allocations for two specific uplink multi-access channels: information theoretic AWGN MAC channel, and CDMA-based uplink.**

***Index Terms*— Game theory, implementation in dominant strategies, mechanism design, rate control, utility maximization**

I. Introduction

As the demand for broadband wireless data services grows, it becomes increasingly necessary to re-examine system design with respect to resource allocation. Wireless spectrum is an inherently shared and limited resource, making efficient resource allocation a crucial aspect of any network design. One of the difficulties in achieving this goal is due to the fact that the definition of what constitutes an efficient resource allocation often depends on private information held by the end-users. For example, we may be interested in minimizing delay and queue backlogs in the system, but in an uplink scenario, the queue backlog is private information known only to the user. This problem is even more pronounced in the case of QoS delivery in networks where data, voice, and video traffic coexist. Similarly, we may be interested in maximizing the aggregate utility of the users, or we may be interested in a trade-off between fairness and priority for different users while users hold key information needed to establish a "socially optimal" solution. The common thread in all of these scenarios is the fact that the network needs to solicit privately held information about the end-users in order to determine whether a particular rate allocation is efficient.

In a scenario where users hold private information but are well-behaved (meaning they are interested in helping the network achieve an efficient allocation), then the network can simply ask the users for their private information. However, in realistic scenarios where users are selfish and interested in maximizing their own utility (known as strategic users), users may have an interest in misrepresenting their private information for their own benefit - even at the expense of overall network efficiency [1]-[3]. Thus, we need to develop a mechanism that creates an incentive for truth telling, ensuring that users have interest to truthfully reveal their private information.

The concept of incentive compatible mechanism design has long been studied, and applies to a wide array of social utility problems. In economics literature, an elegant way to address this problem is the introduction of a *numeraire commodity* - a commodity for which users have extremely large or infinite demand (e.g. money). Users are charged (in terms of this numeraire) for their consumption of the original commodity of interest in such a way that their overall utility is maximized when they behave in a socially responsible manner [4]. A common way to address this problem in both wired and wireless networks is to use dollar-valued pricing schemes in which money is modeled as the numeraire commodity, and users pay for service (see [5]-[8]). Such schemes may, however, require the implementation of pricing schemes where prices are updated at the rate of, for example, channel variations or changes in network topology. An alternative approach used for wireless networks is to model an explicit energy cost to the users (see [9]-[11]).

Taking an approach similar to the dominant philosophy in the peer-to-peer networking paradigm [12], [13], the goal of our paper is to develop mechanisms where the numeraire commodity is the downlink rates. We use the inherent asymmetry between uplink and downlink and (in terms of control and demand) to model the downlink bandwidth as a numeraire commodity. In particular, we use a centralized downlink scheduler to construct an incentive compatible mechanism to ensure a socially efficient use of the uplink. We show that with an appropriately designed downlink scheduler the socially optimal uplink rate allocation emerges as a dominant strategy for all users. To the best of our knowledge, this is a novel approach which has not previously been studied in the context of cellular communication.

In [14] and [15], we have shown first steps in using a centralized downlink scheduler to regulate uplink rate assignments. Specifically, [14] addresses the tragedy of commons problem for distributed CDMA. We show that it is possible to construct a game in which the socially optimal rate allocation arises as a subgame perfect Nash equilibrium. Users in [14], however, are considered to be price-takers; that is, they do not understand or exploit their own impact on the resulting downlink allocation. We address this issue of strategic

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(i.e. non-price-taking) users in [15], where we construct an incentive-compatible mechanism that is robust to strategic behaviors of users. However, the mechanism presented in [15] was designed specifically for an information-theoretic MAC uplink and time-division downlink channel pair. In this paper we extend these results to general (possibly non-convex) uplink rate regions and general convex and coordinate convex downlink regions. In addition, we introduce machinery to mathematically evaluate the performance of our mechanism in terms of downlink performance, and address distributed control of uplink rates.

The rest of this paper is organized as follows. Section II introduces problem formulation, while Section III introduces the joint rate allocation mechanism which is the focus of this paper. Section IV provides numerical results on the performance of the resulting rate allocations for two specific multi-access channels: an information theoretic AWGN MAC channel, and a CDMA-based channel. Section V addresses the issue of distributed rate allocation, and introduces a distributed version of the joint rate allocation mechanism for the CDMA-based channel. Finally, Section VI gives our conclusions and areas of future work.

II. Socially Optimal Rate Allocations

In this paper, we consider uplink rate assignments $(\alpha_1, \ldots, \alpha_N)$ for a single-cell network with N users. The goal is to achieve a socially optimal uplink rate allocation; that is, the uplink rate allocation that maximizes the total utility of the users in the network. Let the function $U_i(\alpha_i) = \theta_i u(\alpha_i)$ be the utility that each user i receives from rate α_i, where $\theta_i \in [0,1]$ is a user-specific constant [1]. The function $u(\cdot)$ is a known and fixed function that is common to all users, and referred to as the homogenous component of the utility function. This homogenous component of each user's utility satisfies the following properties:

$$u(0) = 0 \tag{1}$$

$$u'(\alpha_i) \geq 0 \tag{2}$$

$$u''(\alpha_i) \leq 0 \tag{3}$$

These properties essentially guarantee that users' uplink utilities satisfy diminishing returns.

Finally, let Δ_U be a bounded region of feasible uplink rates. The socially optimal uplink rate allocation problem can be written as:

<u>Problem (P)</u>

$$\max_{\underline{\alpha} \in \Delta_U} \sum_{i=1}^{N} \theta_i u(\alpha_i)$$

A. Inefficiency of Dominant Strategy Equilibrium

Solving Problem (P) in a centralized manner at the base station is straightforward as it is written. Consider, however, the situation where θ_i is private information held by the user, and unknown to the base station. The constant θ_i is referred to as user i's *private type*. The base station must trust the users to reveal their type in a truthful manner. Users, however, are interested in maximizing their own utility and will misrepresent their type if it is to their benefit to do so. We can formulate this scenario as a non-cooperative rate assignment game, where users report values $w_i \in [0,1]$ and the base station allocates rates according to $\max_{\underline{\alpha} \in \Delta_U} \sum_{i=1}^{N} w_i u(\alpha_i)$. It is easy to see that it is a dominant strategy for all users to report $w_i = 1$, regardless of their true value of θ_i. The result is an uplink rate allocation that does *not* solve Problem (P).

This problem is an example of a well known economic problem, where the goal is to achieve *preference revelation* for the allocation of a public good whose utility to users exhibits a diminishing returns property. In other words, the goal is to align the interests of the users with that of the social utility, so that it is in the best interests of the users to reveal their private information in a truthful manner. The most common way to achieve this is to introduce a second commodity into the model, where users are charged for their use of the original commodity in terms of the second commodity. Typically, this second commodity is money. In the next section, we examine the use of an alternative commodity: downlink rate.

B. Leveraging Downlink Rates for Efficient Preference Revelation

In order to solve Problem (P) when users have private information, we take advantage of the users' interest in their assigned downlink rates - a commodity whose allocation is in the hands of the base station, and whose value to the users is assumed to be of a linear form. Recall that on the uplink, we assumed the form of users' utility functions were public knowledge, but were parameterized by a value privately held by the users. On the downlink, we again assume that the form of users' utility functions are public knowledge; however, we assume that the value parameterizing the downlink utility is also public knowledge. Thus, there is no need for preference revelation where downlink utilities are concerned.

Let A_i be the downlink rate allocated to user i. Then the total utility of user i can be expressed as:

$$V_i(\alpha_i, A_i) = \theta_i u(\alpha_i) + A_i \tag{4}$$

Utility functions of this form are referred to as quasi-linear, where the uplink rate is known as the *commodity of interest*, and the downlink rate is known as the *numeraire commodity* [4], [16], [17]. Of course, the choice of utility function is a question of modeling. The utility given by (4) has two important features: the asymmetry of demand between uplink and downlink, and the asymmetry of information between uplink and downlink. Here, we attempt to motivate our modeling choice with respect to these two features.

First we note that in two-commodity markets, quasi-linear utility functions model the fact that users have a much greater demand for one good than another. This is in accordance with the philosophy driving the design of realistic communications systems that can sustain higher rates on the downlink than on the uplink - that users have a greater demand for downlink capacity than for uplink. For example, under full buffer assumptions and using dual receive antennas, cdma1xEVDO Rev

[1]The range of θ_i can easily be extended to any non-negative, finite range

A systems have sector capacity of 1500kb/s on the downlink, but only 500kb/s on the uplink [18]. Similarly, asymmetric DSL (ADSL) systems are designed to support up to 6.1 Mbps on the downstream channel, but only up to 832 kbps on the duplex channel [19]. We believe that these are indications that users have a higher demand for downlink rate as compared to uplink rate, making the choice of modeling downlink as a numeraire commodity for uplink a reasonable one.

Next, consider the asymmetry of information between uplink and downlink. We assume that the *form* of both uplink and downlink utility is known to the base station. In other words, we assume that the base station knows there is an inherent difference in users' demand for uplink and downlink (a reasonable assumption given the discussion above). The asymmetry of information comes from the fact that the uplink and downlink are linearly parameterized by a constant (θ_i on the uplink, 1 on the downlink), where the base station knows the downlink parameter but does not know the uplink parameter.

It turns out that such an assumption on asymmetry of information has realistic interpretations in practical networks as well. Consider, for example, the application of video transmission. There is growing body of work in which transmission power and rate are adapted in response to the needs of the video encoder. For example, the authors in [20] examine the joint adaptation of source coding parameters and power and rate adaptation to minimize transmission energy subject to delay constraints for a single user. It turns out that decisions about video encoding and rate adaptation are, in part, based on the video encoder's buffer delay. The authors in [21] consider scheduling and rate adaptation for video streaming in multi-user wireless networks. Here, the authors present a multi-objective optimization problem, where the maximum buffer length of each user is constrained. Again, the buffer size of each user becomes a critical parameter in deciding both the rate adaptation and transmission schedule for users.

Since such information about the uplink queue backlog or buffer sizes is locally available only to the user, the base station must solicit this information from the users. Information about the downlink queue backlog, however, is locally available to the base station (since downlink queues are maintained at the base station). This exactly motivates our model of asymmetric information about the utility function parameters if we consider θ_i to be ratio of uplink queue backlog to buffer space. As a user's buffer becomes closer and closer to filling, the probability of dropping a packet due to buffer overflow increases. As users become more likely to drop packets, their valuation of uplink rate increases since they can no longer be patient in having their packets served. Choosing θ_i to be the ratio of a user's queue backlog to buffer size captures this effect - it differentiates the uplink utility of users based on their probability of packet loss. Furthermore, recall that we have modeled users' downlink utility as a linear function with parameter 1. This comes from the fact that in systems where users have a much greater demand for downlink than for uplink, all users see long downlink queue backlogs - there is little differentiation of users' downlink utilities based on probability of packet loss[2].

C. Problem Formulation

In the remainder of this paper, we use the quasi-linear preferences of users to design a mechanism (or set of rules through which the users interact) to induce an efficient dominant strategies equilibrium for Problem (P). In other words, we allow the base to allocate a vector of downlink rates $\underline{A}$ in response to the users' announcement of their private types. The problem of allocating socially optimal uplink rates in the presence of private information then reduces to the design of a set of rules of interaction, described mathematically by a mechanism.

Intuitively, any mechanism consists of three main stages. First, the *leader* (in this case the base station) announces the rules of the mechanism (known as an *outcome function*). Next, the *agents* (in this case the mobiles) submit information about their private type to the leader (known as *messages*). Finally, the leader allocates resources based on the information obtained from users in the second stage of the mechanism and the rules announced in the first stage of the mechanism. The goal of the mechanism is to ensure an allocation of the commodity of interest based on agents' private types, despite the fact that the true value of these types may be known only by the agents themselves.

Mathematically, mechanisms are described as a pair (M, g). Let M_i be the message space for user i, and $M = M_1 \times \ldots \times M_N$ be the cross-product of message spaces. In addition, let $g : M \to D \times R^{(N+1)}$ be an *outcome function* that maps messages to outcomes, where an outcome consists of a *decision rule* $D(\cdot)$ and *transfer functions* $t_i(\cdot),\ i = 1, \ldots, N$. The decision rule specifies the allocation of the commodity of interest (in our case the uplink rate), and the transfer functions specify the amount of numeraire commodity allocated to each user (in our case the downlink rate). For a more detailed discussion of mechanism design, see [4], [17], [16].

The area of mechanism design has been well studied in the economic literature - there are known mechanisms that can be used to implement Problem (P). In an economic context, however, mechanisms make no distinction between how much a user values a resource, and how much a user can afford to pay for it. One can think of users as having an unlimited credit card - as long as a user's value for the resource exceeds its payment for the resource, the user is satisfied. Unfortunately, this interchangeability of value and payment breaks down when we consider allocating physical quantities (i.e. rates) in a communication network. In particular, downlink rates are usually restricted to a set Δ_D, whose structure is dependent on channel characteristics as well as physical layer design. The main challenge in applying known mechanisms in this context is the physical infeasibility of allocating non-positive or unrestricted downlink rates. In economic terms, this means we need to design a mechanism for users with finite budgets.

[2] It is fairly straightforward to extend our results to the situation where users' downlink utilities are still linear, but parameterized by a value β_i, where $\underline{\beta}$ is lower bounded and known at the base station.

To our knowledge, the study of such mechanisms has been largely neglected.

As we have seen, the goal of this work is to achieve a socially optimal uplink rate allocation in the presence of private information and strategic users (i.e. to implement Problem (P) in dominant strategies). We have seen that by leveraging the users' interests in downlink rate assignments, this problem reduces to the design of a (fined budget) mechanism where users are charged in terms of downlink for their consumption of uplink. We now introduce the formal problem statement we intend to solve:

Problem (P1)

Design a set of rules $\underline{\alpha}^* = D(\underline{w})$ and $A_i^* = t_i(\underline{w})$ such that for any vector of private types $\underline{\theta}$ and any vector of messages $\underline{w}$, we have:

1. $D(\underline{\theta}) = \arg\max_{\underline{\alpha}\in\Delta_U} \sum_{i=1}^N \theta_i u(\alpha_i)$
2. $(t_1(\underline{w}), \ldots, t_N(\underline{w})) \in \Delta_D$
3. $$\begin{aligned} \theta_i &= \arg\max_{w_i\in[0,1]} [\theta_i u(D_i(w_i, w_{-i})) \\ &+ \ t_i(w_i, w_{-i})] \ \forall \ i \end{aligned}$$

Note that 1) ensures the uplink rate allocation is socially optimal, while 2) ensures the feasibility of downlink allocations. Finally, 3) establishes truth-telling as a dominant strategy, i.e. the users will maximize their own individual utility by truthfully revealing their private type θ_i, regardless of the actions of other users.

III. Joint Rate Allocation Mechanism

We now describe the *Joint Rate Allocation Mechanism* (JRAM).

1) *Message Space* - Each user selects a single value $w_i \in [0,1]$ as its message. In other words, $M_i = [0,1] \ \forall i$, and $M = [0,1] \times \ldots \times [0,1]$.

2) *Decision Rule* - The base station will assign uplink rates $\underline{\alpha}^*$ according to the following rule:

$$\underline{\alpha}^* = D(\underline{w}) = \arg\max_{\underline{\alpha}\in\Delta_U} \sum_{i=1}^N w_i u(\alpha_i) \tag{5}$$

where $u(\alpha_i)$ is the diminishing returns portion of the uplink utility, which is homogenous across all users and known at the base.

3) *Transfer Functions* - The base station will assign downlink rates $\underline{A}^*$ according to the following rule:

$$A_i^* = t_i(\underline{w}) = C_i + \sum_{j\neq i} w_j u(\alpha_j^*) - \sum_{j\neq i} w_j u(\alpha_j^i) \tag{6}$$

where $\underline{\alpha}^i$ is the solution to

$$\underline{\alpha}^i = \arg\max_{\underline{\alpha}\in\Delta_U} \sum_{j\neq i} w_j u(\alpha_j) \tag{7}$$

and $\underline{C}$ is the fixed proportional fair downlink rate allocation - i.e. is the solution to

$$\underline{C} = \arg\max_{\underline{C}'\in\Delta_D} \sum_{i=1}^N \log(C_i') \tag{8}$$

In the first stage of the mechanism, users report their private types as w_i. In the second stage of the mechanism, the base station assigns uplink rates *as if the users reported* $w_i = \theta_i$. In other words, the base station assumes users told the truth, and allocates the uplink rates accordingly. In the final stage of the mechanism, each user is allocated an initial downlink rate corresponding to its proportional fair rate allocation. Users are then "charged" from that initial allocation based on their impact on the system, given by $\sum_{j\neq i} w_j u(\alpha_j^*) - \sum_{j\neq i} w_j u(\alpha_j^i)$.

A. Analysis of the Mechanism

In most cases, the goal of mechanism design is simply to ensure a socially optimal allocation of the commodity of interest. As previously mentioned, however, since the numeraire commodity in the joint rate allocation mechanism is a physical quantity (i.e. downlink) there is an additional concern - namely, ensuring that the resulting downlink rate allocations are feasible and non-negative.

To see how infeasible downlink rate assignments can arise, consider the following example. We have a network of $N = 5$ users with homogenous utility component $u(\alpha_i) = \log(1+\alpha_i)$, and private information $\theta_1 = \theta_2 = .5$, $\theta_3 = \theta_4 = \theta_5 = 1$. The uplink feasible rate region is $\Delta_U = \{\underline{\alpha} : \sum_{i=1}^N \alpha_i \leq 1, \ \alpha_i \leq .5 \ \forall i\}$, and the downlink feasible rate region is $\Delta_U = \{\underline{\alpha} : \sum_{i=1}^N \alpha_i \leq 1\}$. If users truthfully report their values, the mechanism operates as follows:

1. Users report values $\underline{w} = (.5, .5, 1, 1, 1)$
2. Uplink rates are assigned according to (5), giving $\underline{\alpha}^* = (0, 0, .33, .33, .33)$
3. Downlink rates are assigned according to (6). More specifically, from (7) we have

$$\underline{\alpha}^1 = \underline{\alpha}^2 = (0, .33, .33, .33)$$
$$\underline{\alpha}^3 = \underline{\alpha}^4 = \underline{\alpha}^5 = (0, 0, .5, .5)$$

and from (8) we have

$$\underline{C} = (.2, .2, .2, .2, .2)$$

This gives downlink rate assignments $\underline{A}^* = (.2, .2, -.0356, -.0356, -.0356)$.

We see that users 3-5 are assigned infeasible (negative) rates. Clearly we cannot allocate negative rates, nor can we simply assign these users a downlink rate of '0' without compromising the incentive compatibility of the mechanism. The question, then, is how can we guarantee that the transfer functions fall into a given feasible rate region? The approach we have taken is to require an extra technical condition by which the total number of users are kept below a fixed value N_0. We will see in Section IV that this condition does not seriously impact the design.

We now introduce the following theorem, for which the supporting lemmas can be found in Appendix I.

Theorem 1: Assume that Δ_U is bounded, and that Δ_D is bounded, convex, and coordinate convex. Then $\exists \ N_0(\Delta_U, \Delta_D, u(\cdot))$ such that $\forall \ N \leq N_0$, the joint rate allocation mechanism (JRAM) solves Problem (P1).

Proof:

That the joint rate allocation mechanism satisfies Condition 1 is obvious by construction. To see that it satsifies Condition 3, recall that the total utility of user i is written as

$$V_i(\alpha_i, A_i) = \theta_i u(\alpha_i(\underline{w})) + A_i(\underline{w})$$

If the joint rate allocation mechanism does not satisfy Condition 3, then there exists at least one user i and a value $\hat{w}_i \neq \theta_i$ such that

$$\theta_i u(\alpha_i(\hat{w}_i, w_{-i})) + A_i(\hat{w}_i, w_{-i}) > \theta_i u(\alpha_i(\underline{w})) + A_i(\underline{w}) \quad (9)$$

Let $\underline{\hat{\alpha}} = D(\hat{w}_i, w_{-i})$ be the solution to the maximization problem when user i bids $\hat{w}_i$. We can rewrite (9) as

$$\begin{aligned}\theta_i u(\hat{\alpha}_i) \quad &+ \quad C_i + \sum_{j\neq i} w_j u(\hat{\alpha}_j) - \sum_{j\neq i} w_j u(\alpha_j^i) \\ &> \quad \theta_i u(\alpha_i^*) + C_i + \sum_{j\neq i} w_j u(\alpha_j^*) - \sum_{j\neq i} w_j u(\alpha_j^i)\end{aligned}$$

which reduces to

$$\theta_i u(\hat{\alpha}_i) + \sum_{j\neq i} w_j u(\hat{\alpha}_j) > \theta_i u(\alpha_i^*) + \sum_{j\neq i} w_j u(\alpha_j^*) \quad (10)$$

However, since by definition

$$\underline{\alpha}^* = \arg\max_{\underline{\alpha}\in\Delta_U} \sum_{j=1}^N w_j u(\alpha_j) = \arg\max_{\underline{\alpha}\in\Delta_U} \sum_{j=1}^N \theta_j u(\alpha_j) \quad (11)$$

then (10) is clearly a contradiction. Hence, the joint rate allocation mechanism satisfies Condition 3. Note that since the joint rate allocation mechanism is an extension of a VCG mechanism, the preceding proof is a simple extension of the known proof of incentive compatibility for VCG mechanisms (see e.g. [4], [16], [17]).

That the joint rate allocation mechanism satisfies Condition 2 requires several definitions and lemmas which are stated below. The proofs for these lemmas can be found in Appendix I. The outline for this portion of the proof is as follows:

1. Any downlink feasible region Δ_D will at least contain the region Δ_{TDM} achieved by time-sharing between the maximum individual feasible rates (Lemma 1).
2. The proportional fair downlink rate allocation for Δ_D is (component-wise) greater than or equal to the proportional fair downlink rate allocation for Δ_{TDM} (Lemma 3).
3. With an appropriate choice of N_0, the admission control policy $N \leq N_0$ guarantees that each user's "penalty" is less than the proportional fair downlink rate allocation for Δ_{TDM}, hence is less than the proportional fair downlink rate allocation for Δ_D. This guarantees the downlink rates are all non-negative (Lemma 4).
4. By construction, the assigned downlink rates will be less than the proportional fair rate allocation for Δ_D. Since Δ_D is assumed to be a coordinate-convex region, the downlink rates fall within Δ_D (Lemma 4).

The formal proof is as follows.

Definition 1: We denote by $\underline{a}$ the maximum allowable downlink rates for each user - i.e. $a_i = \max\{A_i : A_i \in \Delta_D\}$.

Definition 2: We denote by $\Delta_{TDM} = \{\underline{A} \geq 0 : \sum_{i=1}^N \frac{A_i}{a_i} \leq 1\}$ the region achieved by time sharing between the endpoints $(\hat{a}_1^i, \dots, \hat{a}_N^i)$, where $\hat{a}_j^i = \begin{cases} a_i & \text{if } j = i \\ 0 & \text{else} \end{cases}$.

Lemma 1: $\Delta_{TDM} \subseteq \Delta_D$.

Lemma 2: Let $\underline{\hat{C}}$ be the proportional fair downlink rate allocation for Δ_{TDM}. Then $\underline{\hat{C}}$ is given by:

$$\hat{C}_i = \frac{a_i}{N} \ \forall \ i \in \mathcal{N} \quad (12)$$

Lemma 3: Let $\underline{\hat{C}}$ be the proportional fair downlink rate allocation for Δ_{TDM}, and $\underline{C}$ be the proportional fair downlink rate allocation for Δ_D. Then $\underline{C} \geq \underline{\hat{C}}$.

Lemma 4: The downlink rate allocation $A_i^* = t_i(\underline{w})$ described by (6) is feasible (in other words, $(A_1^*, \dots, A_N^*) \in \Delta_D$).

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B. Budget Imbalance - The Cost of Preference Revelation

Up to this point, the only performance issue we have discussed with respect to the downlink has been the feasibility of the rate allocations. This is because our goal in this paper is to design an optimal uplink rate assignment mechanism - the downlink is used strictly as a regulatory tool. Notice, however, that the construction of the downlink rate assignments given by (6) is such that, unless all users are limited only by their individual maximum rate and not by the multiple access constraints, the downlink rate allocation is guaranteed not achieve a proportional fair (or even pareto optimal) downlink allocation. In fact, it is known that there is no mechanism that achieves both incentive compatibility (preference revelation) of the uplink rates and "efficient" downlink rates (known in economic literature as *budget balance*) for a finite number of users (Proposition 23.C.6 in [4]). That being said, it is obviously undesirable to achieve incentive compatibility of the uplink allocation at the expense of the downlink allocation.

In order to study the performance of the joint rate allocation mechanism with respect to downlink allocations, we draw on terminology from non-cooperative game theory. Often, the equilibrium points that arise from a non-cooperative game are sub-optimal with respect to overall system performance. One way to quantify this sub-optimality is the *price of anarchy* [22], defined as:

$$\rho_A = \frac{\text{system utility of the worst equilibrium}}{\text{optimal system utility}}$$

In other words, the price of anarchy characterizes the loss in overall system utility that is caused by the non-cooperative behavior of users.

In contrast, the equilibrium of a game induced by a social utility mechanism[3] is, by definition, an optimal allocation of the commodity of interest. Instead, the inefficiency of such a game comes from the transfer functions - i.e. in the allocation

[3] A mechanism in which the goal of the system is to maximize the aggregate utility of the user.

of the numeraire commodity. Similar to the price of anarchy, we define a measure which we call the *cost of preference revelation* as follows:

$$\rho_{PR} = \frac{\text{system utility of the actual downlink allocation}}{\text{optimal system utility}}$$

Recall that the initial downlink allocation defined by (8) was a proportional fair allocation (the allocation that maximized the sum of the log of the rates). With that in mind, we can define the utility of any downlink allocation $\underline{A}$ *to the system* as $\sum_{i=1}^{N} \log(A_i)$,[4] and the cost of preference revelation as

$$\rho_{PR} = \frac{\sum_{i=1}^{N} \log(A_i^*)}{\sum_{i=1}^{N} \log(C_i)}$$

where $\underline{A}^*$ is the downlink allocation given by the joint rate allocation mechanism, and $\underline{C}$ is the proportional fair rate downlink rate assignment for the appropriate downlink feasible region. In other words, the cost of preference revelation characterizes the loss in overall system utility of the downlink that is caused by the enforcement of truthful preference revelation.

In the following section, we will examine the cost of preference revelation for two particular uplink feasible regions: an information-theoretic AWGN MAC channel, and a CDMA-based channel.

IV. Numerical Results

The formulation and results presented in Section II and III apply to a wide class of wireless networks. In fact, the only restrictions posed on the uplink and downlink feasible rate regions are that they are bounded, and that the downlink region is convex and coordinate convex. Recall from Theorem 1, however, that there exists some $N_0(\Delta_U, \Delta_D, u(\cdot))$ such that when $N \leq N_0$, the joint rate allocation mechanism is shown to solve Problem (P1). In other words, we impose an admission control criteria that limits the number of users in the system, and is a function of the uplink and downlink feasible rate regions as well as the uplink utility functions.

Here, we formally define the admission control policy. The justification for the choice of N_0 can be found in the proof of Theorem 1 in Appendix I.

Definition 3: The *admission control policy* for the joint rate allocation mechanism is $N \leq N_0(\Delta_U, \Delta_D, u(\cdot))$, where

$$N_0 = \frac{\min_i a_i}{u(\max_i \alpha_i^{max})} \tag{13}$$

and $\alpha_i^{max} = \max\{\alpha_i : \alpha_i \in \Delta_U\}$, and $a_i = \max\{A_i : A_i \in \Delta_D\}$.

In this section of the paper, we examine the performance of the joint rate allocation mechanism in greater detail for two specific uplink regions: an information-theoretic AWGN MAC channel, and a CDMA-based channel. For the downlink in both cases, we work with the standard feasible region associated with a Gaussian broadcast channel using time-division and fixed transmission power. The result is the following feasible rate region, whose derivation is found, for example, in Chapter 14 of [23].

Definition 4: The *downlink feasible rate region* Δ_D^{TDM} is the set of rates $\underline{A}$ that satisfy the following conditions:

C1. $A_i \geq 0 \ \forall \ i$
C2. $\sum_{i=1}^{N} \frac{A_i}{W \log(1+\frac{P_0 G_i}{N_t W})} \leq 1$

A. AWGN Multi-Access Channel

In this subsection, we work with information-theoretic notions of capacity. Each user has an available uplink transmit power p_i, and symmetric uplink/downlink channel gains G_i. The base station has an available downlink transmit power P_0. W is the available bandwidth, and N_t is the thermal noise.

On the uplink, we work with the standard multiple access region in an AWGN channel, defined below. (See Chapter 14 of [23] for the derivation of this result).

Definition 5: The *uplink feasible rate region* Δ_U^I is the set of rates $\underline{\alpha}$ that satisfy the following conditions:

C1. $\alpha_i \geq 0 \ \forall \ i$
C2. $\sum_{i=1}^{N} \alpha_i \leq W \log(1 + \frac{\sum_{i \in S_k} p_i G_i}{N_t W}) \ \forall \ k$

where $\mathcal{S} = \{S_1, \ldots, S_k\}$ is the set of all possible combinations of users.

B. CDMA-Based Channel

In this section, we consider a CDMA-based uplink channel designed for data-optimized (DO) networks, similar to that of cdma 1xEVDO. In such a network, the definition of uplink feasible rates depends on both a target $\frac{E_b}{N_t}$ (denoted by γ) and a target interference level (denoted by K). A more detailed explanation of these feasibility criteria can be found in the 3GPP2 standards for CDMA2000 [24].

An important (and often neglected) issue in high data rate CDMA networks is the performance degradation due to multipath interference when low spreading gains are used [25], [26]. While the standard Gaussian approximation used for performance analysis of a matched filter receiver is valid for high spreading gain, it becomes less and less valid as the spreading gain decreases. As such, we restrict our attention to uplink transmission rates that satisfy $\alpha_i \leq \frac{W}{4}$. This results in a spreading gain which the authors in [25] have shown to exhibit only moderate performance degradation due to multipath interference.

With these issues in mind, we introduce our uplink feasible rate region whose derivation is shown in Appendix II.

Definition 6: The *uplink feasible rate region* Δ_U^C is the set of uplink rates $\underline{\alpha}$ such that

$$\Delta_U^C = \{\underline{\alpha} : \sum_{i=1}^{N} \frac{\alpha_i}{\alpha_i \gamma + W} \leq \frac{K}{\gamma(1+K)}, \ 0 \leq \alpha_i \leq \frac{W}{4} \ \forall \ i\}$$

where $\gamma = 4dB$ and $K = 6dB$ are pre-defined constants.

[4]Here we are distinguishing between the utility of downlink to the user and to system as a whole. The utility of downlink rate to a user is linear, as defined by (4). We are assuming (from the choice of a proportional fair allocation as the initial downlink allocation) that the utility of downlink rate to the system is log of the rate.

C. Comparison of AWGN Multi-Access and CDMA-Based Channels

We use the following setup for examining the performance of our rate allocation mechanism, taken in part from [24] and [27]. We assume utility functions of the form $\theta_i u(\alpha_i)$, where $\theta_i \in [0,1]$ and the homogenous component is $u(\alpha_i) = \log(1+\alpha_i)$. The base station is centered on a 5km x 5km grid, and mobiles are positioned randomly. The results given are averaged over 20 trials of different random layouts. Channel gains are generated using a cost-231 propagation model at 1.9 GHz between the mobiles and bases. The available bandwidth W is 1.2 MHz, and the thermal noise level is -169 dBm/Hz. The transmit power available at individual users (uplink) is 200 mW, while the transmit power available at the base station (downlink) is 15 W. Finally, we assume a link budget (range of serviceable channel gains) of -70 dB to -140 dB.

Recall the admission control policy give by (13). Using the system parameters given above gives the following admission control policy for Δ_U^I and Δ_D^{TDM}:

$$N \leq N_0^{AWGN} = \frac{W \log\left(1 + \frac{P_0 G_{min}}{N_t W}\right)}{u(W \log\left(1 + \frac{P_i G_{max}}{N_t W}\right))} \approx 400$$

Similary, for Δ_U^C and Δ_D^{TDM} we have:

$$N \leq N_0^{CDMA} = \frac{W \log\left(1 + \frac{P_0 G_{min}}{N_t W}\right)}{u(\frac{W}{4})} \approx 725$$

First, notice that our admission control criteria does not present a serious limitation on the working of the system, since the information-theoretic system can admit up to 400 users, while the CDMA system can admit up to 725 users. It is, of course, important to note that using different feasibility regions, physical layer parameters, or utility functions will change the number given by the admission control policy. However, the above results above suggest that for most practical scenarios, the admission control policy is a reasonable one in terms of the number of users admitted to the system.

Having satisfied the admission control policy, we examine the cost of preference revelation - a measure of the optimality of downlink allocations in terms of system utility. Figure 1 shows the cost of preference revelation as a function of the number of users admitted to the system. We see that the optimality of downlink rate allocations begins to decrease even before the admission control criteria is reached. As more users enter the system, they drive up the "price" of uplink allocations, making preference revelation more costly.

It is interesting to notice that the cost of preference revelation is more drastically impacted by the addition of users in the AWGN channel than in the CDMA channel. In order to gain intuition about why this phenomenon occurs, consider a simple system as described above but with $N = 5$ users, where all users have symmetric channel gains. Let $p_i = -\sum_{j \neq i} \theta_i \alpha_j^* + \sum_{j \neq i} \theta_i \alpha_j^i$ be the penalties associated with a given uplink rate allocation. Recall that the cost of preference revelation is defined as

$$\rho_{PR} = \frac{\sum_{i=1}^N \log(A_i^*)}{\sum_{i=1}^N \log(C_i)} = \frac{\sum_{i=1}^N \log(C_i - p_i)}{\sum_{i=1}^N \log(C_i)}$$

Since we use the same downlink feasibility region (hence same $\underline{C}$) when computing downlink rate allocations for AWGN or CDMA uplink regions, it is sufficient to restrict our attention to the penalties associated with each region.

First, consider a system where users have equal priorities - e.g. $\theta_i = .1 \ \forall \ i$. The optimal uplink rates for the CDMA and AWGN regions are $[181, 181, 181, 181, 181]$ kbps and $[300, 300, 300, 300, 300]$ kbps, respectively. The penalties associated with these rate allocations are $[.1, .1, .1, .1, .1]$ kbps, and $[2.2, 2.2, 2.2, 2.2, 2.2]$ kbps. Here we see the first reason that the AWGN system sees a higher cost of preference revelation than the CDMA system - the AWGN users tend to be allocated higher uplink rates, which translates into higher penalties.

Now, consider a system where a single user has a higher priority - e.g. $\theta_1 = .2$, and $\theta_i = .1 \ \forall \ i = 2, \ldots, 4$. The optimal uplink rates for the CDMA and AWGN regions are $[567, 114, 114, 114, 114]$ kbps and $[301, 300, 300, 300, 300]$ kbps, respectively. The penalties associated with these rate allocations are $[.84, .098, .098, .098, .098]$ kbps, and $[2.8, 2.8, 2.8, 2.8, 2.8]$ kbps. Here we notice that the change in user 1's priority caused a large change in the rate allocations of the CDMA system, but hardly had any impact on the rate allocations of the AWGN system. The reason for this has to do with the non-convexity of the CDMA region, combined with the concavity of the utility function $\log(1+\alpha_i)$. The concavity of the log function means that equal-rate solutions are favored. Since the AWGN uplink is convex, the increase in user 1's priority was not enough to overcome this "favoring" of equal-rate solutions - hence we see little change in the rate allocations. The CDMA uplink region is non-convex, however - the increase in user 1's priority combined with increase in "capacity" of a non-equal rate solution is enough to overcome the tendency toward equal-rate solutions.

The change in optimal uplink allocation also leads to a change in penalty functions. We see that the majority of penalty in the CDMA system is incurred by a single user - the penalties of the low priority users have decreased. As the number of users in the system increases, the non-convexity of the uplink CDMA region will continue to ensure that the majority of the penalty is incurred by a small group of users. In the AWGN channel, not only do users continue to see uniform penalties, but these penalties have actually increased due to the discrepancy of a single user's priority. This phenomenon becomes more exaggerated as the number of users in the system grows, and as the uniformity of θ decreases.

What we see here is that, in general, the efficiency of the downlink rate allocations is closely tied the specific physical layer parameters of the uplink, convexity of the uplink capacity region, and the uniformity of the private types $\underline{\theta}$.

V. DISTRIBUTED IMPLEMENTATION

The joint rate allocation mechanism presented in Section III is a centralized allocation scheme in terms of control; users report their weight w_i to the base station, and the base station computes the uplink and downlink rate allocation accordingly. While this is a natural structure for the downlink, a distributed

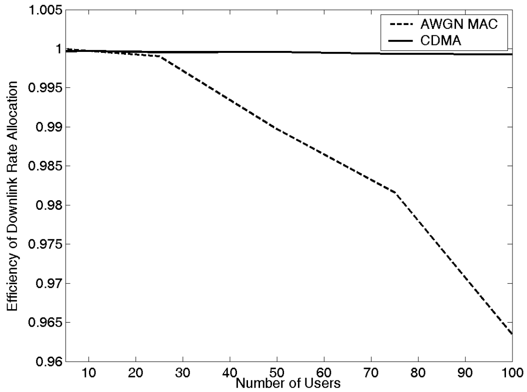

Fig. 1. Efficiency of Downlink Rate Allocations

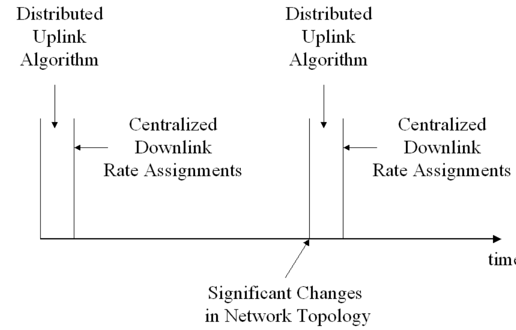

Fig. 2. Relative Time-Scales for Distributed Implementation of the Joint Rate Allocation Mechanism

control structure may be a desirable property on the uplink. In this section, we examine a distributed version of our joint rate allocation mechanism for the CDMA-based channel given in the previous section.

It is well known that the economic notion of pricing can be used to design appropriate signaling mechanisms whose goal is to align the behavior of individual users with the global well being (see e.g. [28]-[34]). In the framework of distributed control, however, agents are considered to be price-takers: that is, they are interested in the goals of the network rather than their own selfish utility. Here there is no notion of private information, and the network is assumed to know (for example) the utility functions of the users. The drawback here is that even when the pricing schemes are implemented using dollar-valued pricing, it has been shown that strategic users can have a significant impact on network efficiency [1]-[3]. Not surprisingly, in cases where pricing schemes are simply signaling mechanisms that do not represent a true cost to the users, the impact of strategic users can be arbitrarily bad in terms of network efficiency, leading to a tragedy of commons situation [3], [6].

In order to develop a distributed implementation of the joint rate allocation mechanism, we consider a two-time scale system, as shown in Figure 2. The base station and mobiles run a distributed algorithm for finding uplink allocations on a fast time-scale. Once this algorithm converges, the base station will assign downlink rates according to (6). As significant changes in topology occur (on a slow time-scale), the process is repeated.

We now introduce the following distributed algorithm for arriving at the uplink rate allocations.

Base Algorithm

The base station announces a regulatory signal μ, which evolves according to the following difference equation:

$$\Delta\mu = \begin{cases} \beta(\sum_{i=1}^{N} \frac{\alpha_i}{W+\gamma\alpha_i} - \frac{K}{\gamma(1+K)}) & \text{if} \quad \mu > 0 \\ \beta[\sum_{i=1}^{N} \frac{\alpha_i}{W+\gamma\alpha_i} - \frac{K}{\gamma(1+K)}]^{+} & \text{if} \quad \mu = 0 \end{cases} \tag{14}$$

where β is a constant.

Mobile Algorithm

Each mobile reacts to the regulatory signal from the base station by adjusting its uplink rate such that

$$\alpha_i = \arg \max_{0 \le \alpha \le \frac{W}{4}} \left(w_i u_i(\alpha) - \frac{\alpha}{W + \gamma\alpha}\mu \right) \tag{15}$$

It has been shown in previous work [33], [34] that the distributed algorithm described by (14)-(15) converges to the solution to Problem P when users choose $w_i = \theta_i$.

Notice that the users are not explicitly reporting w_i to the base station. Instead, the base station waits until the distributed uplink algorithm converges, then computes the weight w_i for each user based on the value of the base's regulatory signal and the uplink rate that each user converged to. Only then does the base station assign downlink rates. This eliminates the possibility for users to manipulate the system by changing weights during the uplink rate assignment process. Each user maximizes its total utility only when its uplink rate converges to the socially optimal allocation.

The two-time scale approach presented here can be extremely useful in practical system design to construct distributed versions of the joint rate allocation mechanism for other physical layer designs while addressing the incentive issues. However, the ability to do so is heavily dependent on the particular uplink physical layer design being considered, and requires combining the results presented in this paper with the particular MAC and physical layer specifications.

VI. Conclusions and Future Work

In this paper, we have constructed a joint rate allocation mechanism that results in feasible uplink and downlink rate assignments. Furthermore, we have shown that by leveraging users' demand for downlink, the mechanism implements the socially optimal rate assignment problem in dominant strategies, even when users act strategically.

Although this paper presents an interesting first result, there is room for future work and improvement.

In constructing the joint rate allocation mechanism, we examine a very specific uplink utility function of the form $U_i(\alpha_i) = \theta_i u(\alpha_i)$. The results presented in this paper can easily be extended to the case where even the function shape of $U_i(\cdot)$ is not known except to the individual users, as long as these utility functions satisfy the same conditions as those given by (1)-(3). However, when considering utility functions that do not have a homogenous component known to the base, the users need to report an entire function rather than a single value. This would pose serious problems in a practical implementation. It would be interesting to examine this work in the context of *scalar VCG mechanisms*, which implement problems in Nash equilibrium (rather than dominant strategies) but only require users to report a single parameter value (see [35], [36]).

Although we have included numerical results regarding the efficiency of downlink allocations, further study on this topic is needed. It is clear, for instance, that the specifics of the uplink physical layer play an important role in the admission control criteria and the resulting downlink allocations. The same can be said of the downlink physical layer, particularly with regard to the modeling of channel gains. Heterogenous channel gains across users results in an imbalance of wealth - users with better channel gains start with a higher proportional fair downlink assignment. One of the functions of the admission control criteria is to ensure that this imbalance of wealth on the downlink does not play a role in how uplink rates are allocated. It will be interesting to study the role of admission control criteria in more complicated scenarios, such as time-varying ergodic channels, or imbalance between uplink and downlink channel gains.

Finally, the mechanism presented in this paper considers incentive-compatible rate allocations for single-cell networks. The extension to multi-cell networks is an important and interesting area for future work. In networks with high-frequency reuse factors where neighboring cells' transmissions are orthogonal, the mechanism presented here can be applied as it is. This is because uplink transmissions cause little or no interference in neighboring cells. As the frequency reuse factor decreases, however, the optimal uplink allocation needs to account for inter-cell interference. This means that in order to preserve incentive-compatibility, each base station needs information about how much interference each user in its cell is contributing *to other base stations* when assigning downlink rates. At first glance, this seems to require complicated message passing schemes between base stations. It will be important to extend the mechanism in this paper not only to account for inter-cell interference, but to do so in a manner with limited overhead.

APPENDIX I

Lemma 1: $\Delta_{TDM} \subseteq \Delta_D$.

Proof: Since Δ_D is a coordinate convex region, we know that the endpoints defined by $(\hat{a}_1^i, \ldots, \hat{a}_N^i)$ are contained in Δ_D. Since Δ_D is a convex region, and Δ_{TDM} is simply the region defined by the convex hull of the endpoints, $\Delta_{TDM} \subset \Delta_D$. ∎

Lemma 2: Let $\underline{\hat{C}}$ be the proportional fair downlink rate allocation for Δ_{TDM}. Then $\underline{\hat{C}}$ is given by:

$$\hat{C}_i = \frac{a_i}{N} \ \forall \ i \in \mathcal{N} \tag{16}$$

Proof: By definition, the proportional fair downlink allocation is the one that solves $\max_{\underline{C} \in \Delta_{TDM}} \sum_{i=1}^N \log(C_i)$ [28]. Since we are maximizing a strictly concave function over a linear region, there is a unique solution which can be found by solving the dual problem [37]. As such, we introduce the LaGrange function:

$$\begin{aligned}\mathcal{L}(\underline{C}, \lambda) &= \sum_{i=1}^N \log(C_i) - \lambda\left(\sum_{i=1}^N \frac{C_i}{a_i} - 1\right) \\ &= \sum_{i=1}^N \left(\log(C_i) - \frac{\lambda C_i}{a_i}\right) + \lambda\end{aligned}$$

We can now formulate the dual problem:

$$\min_\lambda \max_{\underline{C}} \mathcal{L}(\underline{C}, \lambda) = \min_\lambda \max_{\underline{C}} \sum_{i=1}^N \left(\log(C_i) - \frac{\lambda C_i}{a_i}\right) + \lambda \tag{17}$$

Using first-order conditions to solve, we get

$$\lambda^* = \arg\min_\lambda \sum_{i=1}^N \left(\log\left(\frac{a_i}{\lambda}\right) - 1\right) + \lambda = N \tag{18}$$

and

$$\hat{C}_i = \frac{a_i}{\lambda} = \frac{a_i}{N} \tag{19}$$

∎

Lemma 3: Let $\underline{\hat{C}}$ be the proportional fair downlink rate allocation for Δ_{TDM}, and $\underline{C}$ be the proportional fair downlink rate allocation for Δ_D. Then $\underline{C} \geq \underline{\hat{C}}$.

Proof: We do a proof by contradiction. Assume that $\underline{C} = (\hat{C}_i - \Delta C_i, \ldots, \hat{C}_M - \Delta C_M, \hat{C}_{M+1} + \Delta C_{M+1}, \ldots, \hat{C}_N + \Delta C_N)$, where $\Delta C_i \geq 0 \ \forall \ i \in \mathcal{N}$ and $1 \leq M \leq N$. In other words, we assume the proportional fair rate allocation for Δ_D has M components where $C_i \leq \hat{C}_i$.

Let $\underline{\tilde{C}} = (\hat{C}_i - \Delta C_i, \ldots, \hat{C}_M - \Delta C_M, \hat{C}_{M+1} + \Delta\tilde{C}_{M+1}, \ldots, \hat{C}_N + \Delta\tilde{C}_N)$ be the projection of $\underline{C}$ onto Δ_{TDM}. From the definition of $\underline{C}$ as the proportional fair rate allocation for Δ_{TDM}, we have

$$\sum_{i=1}^M \log(\hat{C}_i - \Delta C_i) + \sum_{i=M+1}^N \log(\hat{C}_i + \Delta\tilde{C}_i) \leq \sum_{i=1}^N \log(\hat{C}_i) \tag{20}$$

Now, consider the quantity $\sum_{i=1}^N \log(C_i)$. Using (20), we see that

$$\begin{aligned}\sum_{i=1}^N \log(C_i) &= \sum_{i=1}^M \log(\hat{C}_i - \Delta C_i) + \sum_{i=M+1}^N \log(\hat{C}_i + \Delta C_i) \\ &\leq \sum_{i=1}^N \log(\hat{C}_i) - \sum_{i=M+1}^N \log(\hat{C}_i + \Delta\tilde{C}_i) \\ &+ \sum_{i=M+1}^N \log(\hat{C}_i + \Delta C_i) \\ &= \sum_{i=1}^M \log(\hat{C}_i) + \sum_{i=M+1}^N \log(\hat{C}_i \frac{\hat{C}_i + \Delta C_i}{\hat{C}_i + \Delta\tilde{C}_i})\end{aligned}$$

Since $\underline{C}$ was defined as the proportional fair rate allocation for Δ_D, this implies that the point $\underline{C}' = (\hat{C}_1, \ldots, \hat{C}_M, \hat{C}_{M+1}\frac{\hat{C}_{M+1} + \Delta C_{M+1}}{\hat{C} + \Delta\tilde{C}_{M+1}}, \ldots, \hat{C}_N \frac{\hat{C}_N + \Delta C_N}{\hat{C}_N + \Delta\tilde{C}_N})$ is not contained in Δ_D. We now show that $\underline{C}'$ is, in fact, contained in Δ_D for any value of M such that $1 \leq M \leq N$. This is a contradiction, hence $\underline{C}$ is not the proportional fair rate allocation for Δ_D.

Recall that Δ_D is a convex and coordinate convex region containing $\Delta_{TDM} = \{\underline{A} \geq 0 : \sum_{i=1}^{N} \frac{A_i}{a_i} \leq 1\}$. This region is defined by the axes and a hyperplane with intercepts $(a_1, \ldots, a_N)$.

Now, choose some value L, where $M+1 \leq L \leq N$. Consider the uniquely defined hyperplane that intersects the $N-1$ endpoints $(a_1, \ldots, a_{L-1}, a_{L+1}, \ldots, a_N)$ and the point $\underline{C}$. The equation for this hyperplane is $\sum_{i \neq L} \frac{x_i}{a_i} + \frac{x_L}{b_L} = 1$, where b_L is the L-intercept of the hyperplane and, by construction,

$$\sum_{i=1}^{M} \frac{\hat{C} - \Delta C_i}{a_i} + \sum_{i=M+1, i\neq L}^{N} \frac{\hat{C} + \Delta C}{a_i} + \frac{\hat{C}_L + \Delta C_L}{b_L} = 1 \quad (21)$$

Since $\underline{C} \in \Delta_D$ and Δ_D is convex and coordinate convex, we have $\Delta_D \supseteq \{\underline{A} \geq 0 : \sum_{i \neq L} \frac{A_i}{a_i} + \frac{A_L}{b_L} \leq 1,\ A_L \leq C_L\}$.

Now, let $L = \arg\max_{M+1 \leq i \leq N} \frac{\hat{C}_i}{\hat{C}_i + \Delta \tilde{C}_i}$ and consider $\underline{C}'$. We have

$$\begin{aligned}
\sum_{i \neq L} \frac{C'_i}{a_i} &+ \frac{C'_L}{b_L} \\
&= \sum_{i=1}^{M} \frac{\hat{C}_i}{a_i} + \sum_{i=M+1, i\neq L}^{N} \frac{\hat{C}_i}{a_i} \left(\frac{\hat{C}_i + \Delta C_i}{\hat{C}_i + \Delta \tilde{C}_i} \right) \\
&+ \frac{\hat{C}_L}{b_L} \left(\frac{\hat{C}_L + \Delta C_L}{\hat{C}_L + \Delta \tilde{C}_L} \right) \\
&= \sum_{i=1}^{M} \frac{\hat{C}_i}{a_i} + \sum_{i=M+1, i\neq L}^{N} \frac{\hat{C}_i}{a_i} \left(\frac{\hat{C}_i + \Delta C_i}{\hat{C}_i + \Delta \tilde{C}_i} \right) \\
&+ \frac{\hat{C}_L}{\hat{C}_L + \Delta \tilde{C}_L} \left[1 - \sum_{i=1}^{M} \frac{\hat{C}_i - \Delta C_i}{a_i} \right. \\
&- \left. \sum_{i=M+1, i\neq L}^{N} \frac{\hat{C}_i + \Delta C_i}{a_i} \right] \\
&= 1 - \sum_{i=M+1}^{N} \frac{\hat{C}_i}{a_i} + \sum_{i=M+1, i\neq L}^{N} \frac{\hat{C}_i}{a_i} \left(\frac{\hat{C}_i + \Delta C_i}{\hat{C}_i + \Delta \tilde{C}_i} \right) \\
&+ \frac{\hat{C}_L}{\hat{C}_L + \Delta \tilde{C}_L} \left[1 - \left(1 - \sum_{i=M+1}^{N} \frac{\hat{C}_i + \Delta \tilde{C}_i}{a_i} \right) \right. \\
&- \left. \sum_{i=M+1, i\neq L}^{N} \frac{\hat{C}_i + \Delta C_i}{a_i} \right] \\
&= 1 + \sum_{i=M+1, i\neq L}^{N} \left[-\frac{\hat{C}_i}{a_i} + \frac{\hat{C}_i}{a_i} \left(\frac{\hat{C}_i + \Delta C_i}{\hat{C}_i + \Delta \tilde{C}_i} \right) \right. \\
&+ \frac{\hat{C}_L}{\hat{C}_L + \Delta \tilde{C}_L} \left(\frac{\hat{C}_i + \Delta \tilde{C}_i}{a_i} \right) \\
&- \left. \frac{\hat{C}_L}{\hat{C}_L + \Delta \tilde{C}_L} \left(\frac{\hat{C}_i + \Delta C_i}{a_i} \right) \right] - \frac{\hat{C}_L}{a_L} \\
&+ \frac{\hat{C}_L}{\hat{C}_L + \Delta \tilde{C}_L} \left(\frac{\hat{C}_L + \Delta \tilde{C}_L}{a_L} \right) \\
&= 1 + \sum_{i=M+1, i\neq L}^{N} \left(\frac{\Delta C_i - \Delta \tilde{C}_i}{a_i} \right) \\
&\times \left(\frac{\hat{C}_i}{\hat{C}_i + \Delta \tilde{C}_i} - \frac{\hat{C}_L}{\hat{C}_l + \Delta \tilde{C}_L} \right) \\
&\leq 1
\end{aligned}$$

where the second equality comes from (21), and the third equality comes from the observation that $\underline{\hat{C}}$ and $\underline{\tilde{C}}$ are on the boundary of Δ_D^{TDM} (the fourth and fifth inequalities are just simplification). The final inequality comes from the observation that $\Delta C_i \geq \Delta \tilde{C}_i\ \forall M+1 \leq i \leq N$, and the choice of $L = \arg\max_{M+1 \leq i \leq N} \frac{\hat{C}_i}{\hat{C}_i + \Delta \tilde{C}_i}$.

Furthermore, we note that $C'_L = \hat{C}_L \frac{\hat{C}_L + \Delta C_L}{\hat{C}_L + \Delta \tilde{C}_L} \leq \hat{C}_L + \Delta C_L = C_L$. Hence, $\underline{C}' \in \Delta_D$ and $\underline{C}$ cannot be proportional fair. Since this result holds $\forall\ M,\ 1 \leq M \leq N$, we are done. ■

Lemma 4: The downlink rate allocation $A^*_i = t_i(\underline{w})$ described by (6) is feasible (in other words, $(A^*_1, \ldots, A^*_N) \in \Delta_D$).

Proof:

Note that by definition, $\sum_{j \neq i} w_j u(\alpha^i_j) \geq \sum_{j \neq i} w_j u(\alpha^*_j)$. This gives

$$A^*_i = C_i + \sum_{j \neq i} w_j u(\alpha^*_j) - \sum_{j \neq i} w_j u(\alpha^i_j) \leq C_i\ \forall\ i$$

Since $\underline{C}$ is feasible (by construction) and Δ_D is a coordinate convex region, $\underline{A}^*$ is also be feasible *as long as* $\underline{A} \geq 0$.

In order to show that $\underline{A} \geq 0$, we note the following:

$$\begin{aligned}
\sum_{j \neq i} w_j u(\alpha^i_j) &- \sum_{j \neq i} w_j u(\alpha^*_j) \\
&= \sum_{j \neq i} w_j u(\alpha^i_j) + w_i u(0) - \sum_{j \neq i} w_j u(\alpha^*_j) \\
&\leq \sum_{j=1}^{N} w_j u(\alpha^*_j) - \sum_{j \neq i} w_j u(\alpha^*_j) \\
&= w_i u(\alpha^*_i)
\end{aligned}$$

where the first equality comes from the properties of $u(\cdot)$, and the inequality comes from the definition of $\underline{\alpha}^*$. We also note that

$$w_i u(\alpha^*_i) \leq u(\alpha^*_i) \leq u(\alpha^{max}_i)$$

where the first inequality comes from the definition of the message space as $w_i \in [0,1]$, and the second inequality comes from the boundedness of Δ_U. We now have:

$$\begin{aligned}
A^*_i &= C_i + \sum_{j \neq i} w_j u(\alpha^*_j) - \sum_{j \neq i} w_j u(\alpha^i_j) \\
&\geq C_i - u(\alpha^{max}_i) \\
&\geq \frac{a_i}{N} - u(\alpha^{max}_i)
\end{aligned}$$

where the second inequality comes from Lemma 2 and Lemma 3.

Now, we choose $N_0 = \frac{\min_i a_i}{u(\max_i \alpha_i^{max})} \geq 0$. Using the condition $N \leq N_0$, we have:

$$\begin{aligned} A_i^* &\geq \frac{a_i}{N} - u(\alpha_i^{max}) \\ &\geq \frac{a_i u(\max_i \alpha_i^{max})}{\min_i a_i} - u(\alpha_i^{max}) \\ &\geq u(\max_i \alpha_i^{max}) - u(\alpha_i^{max}) \\ &\geq 0 \end{aligned}$$

and we are done.

■

APPENDIX II

We say a vector of uplink rates $\underline{\alpha}$ is a feasible solution if there exists a vector of transmission powers $(p_1, \ldots, p_N)$ that satisfies the following conditions:

1. $\frac{\frac{W}{\alpha_i} p_i G_i}{N_t W + \sum_{j \neq i} p_j G_j} = \gamma \ \forall i$
2. $\sum_{i=1}^{N} p_i G_i \leq K N_t W$

Rearranging Condition 1 and summing over all users gives:

$$\sum_{i=1}^{N} p_i G_i = \sum_{i=1}^{N} \frac{\gamma(N_t W + \sum_{j=1}^{N} p_j G_j)}{\frac{W}{\alpha_i} + \gamma}$$

Substituting $r_i = \frac{1}{\frac{W}{\alpha_i} + \gamma}$ and $z = \sum_{j=1}^{N} p_j G_j$ yields

$$z = \gamma(N_t W + z) \sum_{i=1}^{N} r_i \tag{22}$$

Notice that, using the above substitution, Condition 2 is nothing but $z \leq K N_0 W$. Thus, by rearranging (22) and substituting into Condition 2, we get the following:

$$\sum_{i=1}^{N} r_i \leq \frac{K}{\gamma(1+K)}$$

Substituting α_i back in gives the feasibility region Δ_U^C.

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