Exercise Set #3

1. Consider any binary tree with leaves at depth \( l_1, \ldots, l_M \). Show that

\[
\sum_{i=1}^{M} 2^{-l_i} \leq 1.
\]

2. Consider an i.i.d. source with alphabet \( \{A, B, C, D\} \) and probabilities \( \{0.1, 0.2, 0.3, 0.4\} \).

   (a) Find a Tunstall code that encodes the source phrases into binary codewords of length 4. Compute the average number of binary code symbols per source symbol and compare it to the entropy (with the appropriate base).

   (b) Now suppose the Tunstall code you found in part (a) is followed by a binary Huffman code. Compute the average number of binary code symbols per source symbol.

   (c) Find a Tunstall code that encodes the source phrases into quaternary code words of length 2. Compute the average number of quaternary code symbols per source symbol and compare it to the entropy (with the appropriate base).

   (d) Suppose the Tunstall code found in part (c) is followed by a quaternary Huffman code. Compute the average number of quaternary code symbols per source symbol and compare it to the entropy (with the appropriate base).