Solutions to Exercise Set #7

1. Draw the convolutional encoders for the following codes:

(a) Rate $\frac{1}{2}$, code with octal tap connections 51 & 63,
(b) rate $\frac{1}{3}$ code with octal tap connections 127 & 135 & 125, and
(c) Rate $\frac{1}{4}$ code with octal tap connections 452 & 542 & 616 & 776.
(d) How many states does the trellis for each encoder have?

Solution:

(a) Since
\[ 51_{\text{oct}} = 101001 \]
\[ 63_{\text{oct}} = 110011, \]
we have
\[ n_1 = 1 + z^{-2} + z^{-5} \]
\[ n_2 = 1 + z^{-1} + z^{-4} + z^{-5}. \]

(b) Since
\[ 127_{\text{oct}} = 001010111 \]
\[ 135_{\text{oct}} = 001011101 \]
\[ 125_{\text{oct}} = 001010101, \]
we have
\[ n_1 = 1 + z^{-2} + z^{-4} + z^{-5} + z^{-6} \]
\[ n_2 = 1 + z^{-2} + z^{-3} + z^{-4} + z^{-6} \]
\[ n_3 = 1 + z^{-2} + z^{-4} + z^{-6}. \]

(c) Since
\[ 452_{\text{oct}} = 100101010 \]
\[ 542_{\text{oct}} = 101100010 \]
\[ 616_{\text{oct}} = 110001110 \]
\[ 776_{\text{oct}} = 111111110. \]
we have

\[ n_1 = 1 + z^{-3} + z^{-5} + z^{-7} \]
\[ n_2 = 1 + z^{-2} + z^{-3} + z^{-7} \]
\[ n_3 = 1 + z^{-1} + z^{-5} + z^{-6} + z^{-7} \]
\[ n_4 = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6} + z^{-7}. \]

(d) We have

\[ (a) : 2^5 = 32 \text{ states} \]
\[ (b) : 2^6 = 64 \text{ states} \]
\[ (c) : 2^7 = 128 \text{ states} \]
2. Consider the following map of roads between Huffmanville and Shannontown. The labels on the branches are mileage.

(a) Using the Viterbi Algorithm starting at the node representing Huffmanville, find the shortest path between Huffmanville and Shannontown.

(b) Using the Viterbi Algorithm starting at the node representing Shannontown, find the shortest path between Shannontown and Huffmanville. Are the paths found in parts (a) and (b) the same?

Solution:

3. Consider the rate 1/2 convolutional encoder with Octal tap connections 13 and 11. Assume that the encoder starts in the all-zero state and that the two outputs are multiplexed into a single output stream.

(a) What is the minimum Hamming weight of any non-zero codeword that starts in the all zero state and ends in the all-zero state?
(b) Draw a trellis diagram that describes the output of the encoder. The trellis should have two binary digits on each branch corresponding to the two outputs of the encoder. Make the trellis large enough so that you show the output sequence for the 5 input (information) digits 10110. What are the output digits for this input sequence?

(c) Assuming the encoded sequence was sent over a binary symmetric with bit error probability less than 1/2, show the branch and state metrics for a Viterbi decoder corresponding to the received binary sequence 0010110011. What is the information sequence corresponding to the best path through the trellis. (If there is a tie, give one of these sequences.)

(d) Assume the encoded sequence was sent over a binary symmetric with bit error probability $p$. Write a computer simulation that yields the decoded bit error probability for the following values of $p = 0.01, 0.05, \text{and } 0.1$.

**Solution:** The minimum Hamming weight can be easily found using the state diagram given in the following diagram. The minimum non-zero weight path from state 000 to state 000 is 5. The trellis diagram is given in the following figure. Using the diagram to encode the input sequence (10110), we get the output, (1100010010). This can also be done using the state diagram. In order to decode the sequence (0010110011), we use the Viterbi algorithm to find the best path through the trellis. The best path is given with dotted lines in the trellis diagram. The decoded bits corresponding to the given path is (00100).