Homework Set #1
(Due: Thursday, April 13, 2017)

Your answer should be as clear and readable as possible. You should not rely on a calculator or computer in solving these problems.

1. For each of the following codes, determine if it is non-singular, uniquely decodable, or instantaneous.
   (a) \{01, 10\}
   (b) \{0, 01, 10\}
   (c) \{0, 10, 11\}
   (d) \{110, 11, 100, 00, 01\}

2. Consider a source which produces an i.i.d. sequence of symbols from the alphabet \{A, B, C\} with probabilities \{0.5, 0.25, 0.25\} respectively. For \(n = 1, 2,\) and \(3,\) find binary Huffman codes for taking \(n\) source symbols at a time. In each case compute the average number of binary code symbols per source symbol and compare the results.

3. Repeat Problem 2 with probabilities \{0.4, 0.35, 0.25\}.

4. Repeat Problems 2 and 3 with Shannon–Fano codes.
Programming Assignment

You can use any programming language you prefer (MATLAB, Python, C/C++, or Julia, for example). Write down your code as clearly as possible and add suitable comments. Turn in the hard copy of answers to each problem and submit your code to ece154ucsd@gmail.com with the exact subject ECE 154C (HW1).

1. Write a program for a function `binaryHuffman(pmf)` that takes a probability vector `pmf` and outputs a binary Huffman code. For example,

   ```
   ["0", "10", "11"] = binaryHuffman([0.8, 0.1, 0.1])
   ```

   (a) Run the program for `pmf = {0.9, 0.1}` and compute the average length of the resulting Huffman code.
   (b) Consider block lengths `n = 2, 5, 10`. For each block length, find the Huffman code and the average length per symbol.
   (c) Repeat (a) and (b) with `n = 1, 3, 5` for probabilities `pmf = {0.5, 0.25, 0.25}`.
   (d) Repeat (a) and (b) with `n = 1, 2, 3` for probabilities `pmf = {0.53, 0.28, 0.1, 0.05, 0.04}`.

2. Write a program for a function `ShannonFano(pmf)` that takes a probability vector `pmf` and outputs a Shannon–Fano code. Repeat the same experiments as in the previous problem.

3. Using the programs in the previous two problems, find a probability `p ∈ [0, 1]` and a block length `n` such that the average length for the Shannon–Fano code for `pmf = {p, 1 − p}` is different from that of the Huffman code.

4. Consider a source with probability vectors `{p, 1 − p}`. We find the average length per symbol when `n = 10` symbols of this source is encoded using Huffman coding.

   (a) Using `binaryHuffman`, plot the average length per symbol. The plot should be evaluated at 100 values of `p`.
   (b) Find the average (over `p`) of the average length per symbol. Comment on how much a Bernoulli source with random bias `P ∼ Unif[0,1]` can be compressed on average.