Hypothesis
- Show To Q 5
- Mention Review

There
- May Be Vars Or A Sum Of These Vars
- Gaussian Random Variable

Mean & Var Of A Sum Of RVs

Let \( X = \{ x_1, x_2, \ldots, x_n \} \)

\[
\]

\[
\text{Var}(Y) = \text{Var}(Y|X) + \text{Var}(X) + 2 \sum \text{Cov}(X_i, X)
\]

If \( X \) \& \( Y \) Are Uncorrelated, Then \( \text{Var}(Y) = \sum \text{Var}(X_i) \)

Ex: A General RV

Let \( X: x_1, x_2, \ldots, x_n \) Be iid B(\( n, p \)) \& \( Y: \frac{1}{n} \sum x_i \)

\[
E(Y) = \frac{1}{n} \sum E(X_i) = np
\]

\[
\text{Var}(Y) = \frac{1}{n^2} \sum \text{Var}(X_i) + np(1-p)
\]

Q: How: Same as Previous Then This Here Is A Binomial Can Be A Maximum

Let \( N \) Be The # Of Trials Who Get Their Own Hat

Find \( E[N] + \text{Var}(N) \)

Let \( X_i \sim \) Geometric

\( \{ 0, \text{Otherwise} \} \)

Thus, \( N = \sum_{i=1}^{\infty} X_i \), Hence \( E[N] = \sum_{i=1}^{\infty} E[X_i] = \frac{1}{p} \cdot 1 \)

\( \text{(Exponential Summation) \#} \)

\[
E[(X_1 - E[X_1])^2] = E[X_1] + E[X_1] = E[0] = E[
\]

\( \text{Var}(N) \) Consider \( \text{Var}(X_i) = \frac{1}{p} \cdot \frac{1}{(1-p)^2} \)

\[ E[X_i] = \frac{1}{(1-p)^2} \]

\[ \text{Var}(X_i) = \frac{1}{p} \cdot \frac{1}{(1-p)^2} \]

\[
\text{Cov}(X_i, X_j) = \begin{cases} \frac{1}{p} & i=j \\ 0 & i \neq j \end{cases}
\]

\[
E[X_i^2] = \frac{1}{p} \cdot \frac{1}{(1-p)^2} + \frac{1}{p} 
\]

\[
\text{Var}(N) = \sum_{i=1}^{\infty} \text{Var}(X_i) + 2 \sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} \text{Cov}(X_i, X_j) \cdot (\frac{1}{2}) + \frac{1}{2} \cdot 1
\]

Ex: (b) Common Collector's Problem

Since A Fails On 1st Roll Then Retry Until Each Of The #s / Through C, Amies At Least Once Let \( N \) Be The # Of Rolls. Find \( E[N] \)

Let \( X_i \) Be The # Of Rolls Needed To Obtain The 1st New #
\[ E[N] = \sum_{n=1}^{\infty} \frac{E[X_n]}{n^2} \]

\[ E[X_n] = \frac{1}{n} \quad \text{Geometric} \left( \frac{1}{n} \right) \]

\[ E[X_1] = \frac{1}{1} \quad \text{Geometric} \left( \frac{1}{1} \right) \]

\[ E[X_2] = \frac{1}{2} \quad \text{Geometric} \left( \frac{1}{2} \right) \]

\[ E[X_3] = \frac{1}{3} \quad \text{Geometric} \left( \frac{1}{3} \right) \]

\[ E[X_4] = \frac{1}{4} \quad \text{Geometric} \left( \frac{1}{4} \right) \]

\[ E[X_5] = \frac{1}{5} \]

\[ E[N] = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} - 1 \]

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**Ex. 3: Spaghetti**

We have a bowl of Spaghetti Strands. We add in 2 (or more) strands each time. The process is continued until there are no ends left.

\[ \text{Let } N \text{ be the number of strands. Find } E[N] \]

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No Matter How The Pieces Are Cut, There Are Exactly 15 Strands Or Fewer, 16 Ends.

\[ \text{Let } X_n = \begin{cases} 1 & \text{if no end is cut} \\ 0 & \text{if end is cut} \end{cases} \]

\[ \text{Let } \{X_n\} \text{ be a sequence of random variables.} \]

\[ E[X] = \sum_{n=1}^{\infty} \frac{1}{2^n} \]

\[ E[X_1] = \frac{1}{2} \]

\[ E[N] = \sum_{n=1}^{\infty} \left( \frac{1}{n} \right) = \frac{\pi^2}{6} - 1 \]

\[ \frac{\pi^2}{6} - 1 \]

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**Generalizing Sum Of N**

1. **Linear Transformation Of A RV**

\[ \text{Let } X_1, X_2, \ldots, X_n \text{ be a RV of } M_n \text{ and } \text{ Cov}(M) \text{, } \text{ Let } A \text{ be a matrix. Then } \text{ if } X = A X \text{ is a linear transformation of } X. \]

\[ E(X) = \sum_{i=1}^{n} A_i = \text{ a constant value} \]

Then \[ Y = \sum_{i=1}^{n} X_i = Y = A X \]

\[ E(Y) = \sum_{i=1}^{n} A_i \text{ is calculated by } \]

\[ A \text{ is a constant matrix, } \text{ and } M = \sum_{i=1}^{n} X_i \]

---

\[ \text{Ex. 4: Add All Elements Of } A \]

\[ \text{of } \text{Var}(M) = \sum_{i=1}^{n} \text{ Cov}(X_i) \text{ is added to } \sum_{i=1}^{n} \text{ Var}(X_i) \]

\[ \text{For } A \text{ with diagonal elements } A_{ii} \]

\[ A_{ii} \text{ is a diagonal matrix, } \text{ and } \sum_{i=1}^{n} A_{ii} \text{ is added to } \sum_{i=1}^{n} \text{ Var}(X_i) \]

---

\[ \sum_{i=1}^{n} A_{ii} = \text{ a diagonal matrix} \]

\[ \text{and } \sum_{i=1}^{n} \text{ Var}(X_i) \text{ is added to } \]

\[ \text{when } A \text{ is a diagonal matrix, } \text{ and } M \text{ is a scalar.} \]

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\[ \text{when } A \text{ is a diagonal matrix, } \text{ and } M \text{ is a scalar.} \]
\[ E[Y] = E[AX] = AE[X] + AM \]

\[ E[(X - E[1])(X - E[1])^T] = E[(A(X - E[1]))(A(X - E[1]))^T] = \sum \left( E[A(X - E[1])(X - E[1])^T] \right) = AE[E(A)(E - E)^T]A^T + A \Sigma A^T \]

The variance of vector notation \(\Sigma\) is not shown!

**Condition 1**

Given \( AX \)

<table>
<thead>
<tr>
<th>( X \times x \times \ldots \times x )^T</th>
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</thead>
<tbody>
<tr>
<td>0 \times \Sigma &gt; 0 ??</td>
</tr>
<tr>
<td>( 0 + \Sigma &gt; 0 )</td>
</tr>
<tr>
<td>( \Sigma &gt; 0 )</td>
</tr>
</tbody>
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*Verify that the Mean is consistent with the Form \( E[A] \) is a function of \( m \) and \( \Sigma \)!

\[ \text{Normal: } X \sim N(\mu, \Sigma) \]

**Theorem**

1. \( E[X] = \mu + \Sigma x^T \)
2. **Univariate** \( E \times \text{Normal} \Rightarrow \text{INDEPENDENT} \)

**Special Case:** If \( X \sim (0, 0)^T \), then \( X, x, \ldots \times x \) are i.i.d. \( N(\mu, \Sigma) \) (given Normal-Gaussian RV)

**Proof:** Let a Characteristic Function of a Random Vector \( X \): A Cov Eq. of A Gaussian RV

**Premise:** Let \( A \), a Characteristic Function of a Random Vector \( X \): A Cov Eq. of A Gaussian RV

**Premise:** Given \( X \sim (0, 0)^T \), then \( X, x, \ldots \times x \) are i.i.d. \( N(\mu, \Sigma) \) (given Normal-Gaussian RV)

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