Homework Set #3

1. Let \((q(x^n))_{n=1}^{\infty}\) be pointwise universal w.r.t. \(P\), and \(u(x^n) \equiv 1/|\mathcal{X}|^n\) be the uniform probability density. Show that \((q'(x^n))_{n=1}^{\infty}\), where

\[ q'(x^n) = \frac{n-1}{n} q(x^n) + \frac{1}{n} u(x^n), \]

is both universal and pointwise universal w.r.t. \(P\).

2. Let \((q(x^n))_{n=1}^{\infty}\) be a sequence of pointwise universal subprobability densities w.r.t. \(P\). Show that its normalized version \((q'(x^n))_{n=1}^{\infty}\), where

\[ q'(x^n) = \frac{q(x^n)}{\sum_{x^n} q(x^n)}, \]

is pointwise universal w.r.t. \(P\).