Puzzle #2: Hat Problems
(To be discussed on October 14, 2011)

1. Hats on a line. Ten prisoners are standing in a line, each wearing a red or blue hat. The color of each hat is determined by a coin toss, with the outcome of one coin toss having no effect on the others. Each prisoner can see the hats of those in front of him, but not his own or of those behind him.

Starting from the back of the line, the inquisitor asks the prisoners one by one the color of their hats. Each prisoner can hear the answers called out by those behind him, but no other communication of any sort is allowed, except for an initial strategy session before the inquisition begins. What is the strategy for the prisoners to maximize the number of correct answers?

2. Hats on a circle. Three players enter a room and a red or blue hat is placed on each person’s head. The color of each hat is determined by a coin toss, with the outcome of one coin toss having no effect on the others. Each person can see the other players’ hats but not his own.

No communication of any sort is allowed, except for an initial strategy session before the game begins. Once they have had a chance to look at the other hats, the players must simultaneously guess the color of their own hats or pass. The group shares a hypothetical $3 million prize if at least one player guesses correctly and no players guess incorrectly.

The same game can be played with any number of players. The general problem is to find a strategy for the group that maximizes its chances of winning the prize.

One obvious strategy for the players, for instance, would be for one player to always guess “red” while the other players pass. This would give the group a 50 percent chance of winning the prize. Can the group do better?

(New York Times, April 10, 2001)