# ELEMENTS OF INFORMATION THEORY WORKSHOP

# COVERFEST 2008

## STANFORD UNIVERSITY MAY 16

$$e^{it}_{2008} = \left(\frac{C}{TM}\right)^{70}$$

$$\Delta W(F,G) \le D(F||G)$$

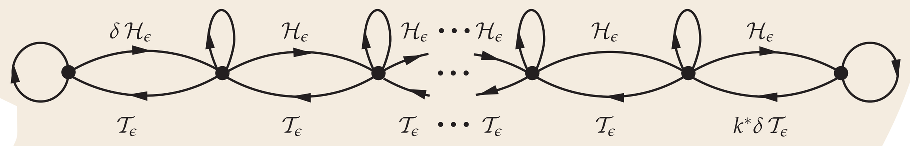
$$\min_{\hat{p}^n\in[0,1]^n}\ \max_{x^n\in\{0,1\}^n} \frac{1}{\sqrt{n}}\left[\sum_{t=1}^{n}|x_t-\widehat{p}_t(x^{t-1})| - \min_{b\in\{0,1\}}\sum_{t=1}^{n}|x_t-b|\right] \longrightarrow \frac{1}{\sqrt{2\pi}}$$

$$R^* = \sup\left[\min\{I(X_1;Y,\hat{Y}_1|X_2,U)+I(U;Y_1|X_2,V),\ I(X_1,X_2;Y)-I(\hat{Y}_1;Y_1|X_2,X_1,U,Y)\}\right]$$

$$R(\ell,u) - R^* = O\left(\frac{1}{u}\right) + \exp\left\{\ell\log\left(2\sqrt{\eta\bar{\eta}}\int\sqrt{f_1(x)f_2(x)}\,dx\right)+o(\ell)\right\}$$


$$\sum_{n=1}^{\infty}\frac{1}{\log^* n} < \infty$$

$$\max_F\left[I(\mathbf{X}+F\mathbf{S};\mathbf{Y})-I(\mathbf{X}+F\mathbf{S};\mathbf{S})\right] = \frac{1}{2}\log\frac{|K_{\mathbf{X}}+K_{\mathbf{Z}}|}{|K_{\mathbf{Z}}|}$$



$$\frac{d}{d\gamma}I(X;\sqrt{\gamma}X+Z) = \frac{1}{2}E\left[(X-E[X|\sqrt{\gamma}X+Z])^2\right]$$

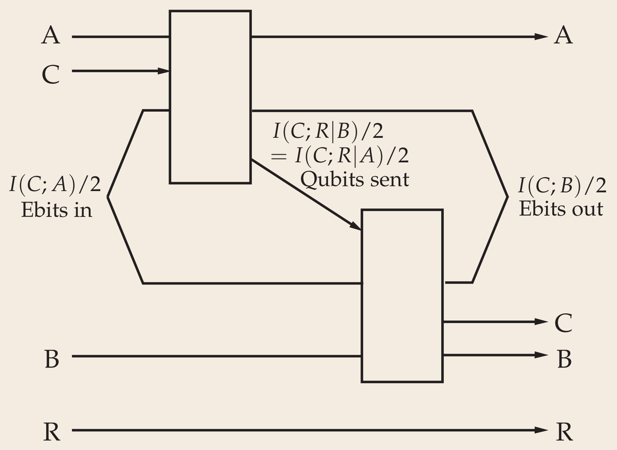
$$i_s(\mathbf{x}) = \sum_{j=1}^{n} x_j\, n_S(x_1,x_2,\ldots,x_{j-1},0)$$

$$\Delta W_{\mathbf{q}} \ge \Delta W_{\mathbf{p}}$$

$$EK(X) \approx H(X)$$

$$\Delta'(0) = \rho_m^2(V,X)$$

$$|P-Q|^2 \le 2D(P||Q)$$




$$C_2 \le 4C_\infty + 3$$

$$H(X_n|X^0_{-\infty}) \nearrow$$

$$dW = WA + \frac{1}{2}\mathcal{L}_W W$$

$$2\log(S^{**}/S^*) \sim \chi^2((m-1)(k-1))$$

$$C = \max_{p(x,u|s)}\left[I(U;S_2,Y)-I(U;S_1)\right]$$

$$W^* = \max_{\mathbf{b}} E\ln\mathbf{b}^t\mathbf{X}$$

$$C\left(\text{channel: }\delta,\ 1-\delta\right) = \log\left(1-\delta^{\frac{1}{1-\delta}}+\delta^{\frac{\delta}{1-\delta}}\right)$$

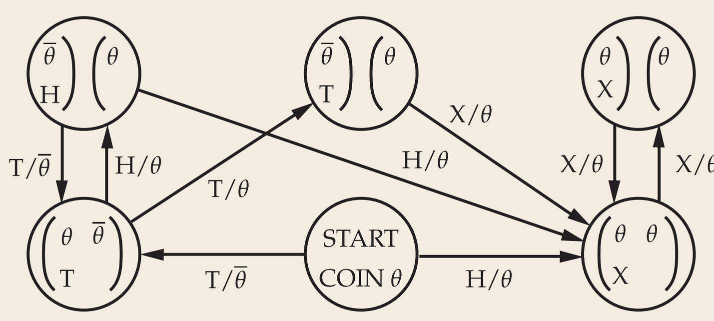
$$\Pr\left\{\left|-\frac{1}{n}\ln f(X_1,\ldots,X_n)-h(X^n)\right|\ge\epsilon\right\}\le e^{-n\left(\epsilon-\frac{1}{2}\ln(1+2\epsilon)\right)}$$

$$C(N,d) = 2\sum_{k=0}^{d-1}\binom{N-1}{k}$$

$$\max_{\hat{\mathbf{b}}}\min_{\mathbf{x}^n}\frac{\hat{S}_n(\mathbf{x}^n)}{S_n^*(\mathbf{x}^n)} = \left[\sum_{n_1+\cdots+n_m=n}\binom{n}{n_1\cdots n_m}2^{-nH\left(\frac{n_1}{n},\ldots,\frac{n_m}{n}\right)}\right]^{-1}$$

$$H^k \searrow H = H^\infty$$

$$\mathbf{b}^* = \mathbf{p}$$




$$b_i' = E\left(\frac{b_iX_i}{\mathbf{b}^t\mathbf{X}}\right)$$

$$e^{-i\pi} = -1$$

$$D(P\,||\,Q) = D(P\,||\,P^*) + D(P^*\,||\,Q)$$

$$\frac{P_{\rm BP}}{1-P_{\rm BP}} = \frac{P_{\rm B}}{1-P_{\rm B}}\frac{P_{\rm P}}{1-P_{\rm P}}\frac{1-P_{\rm L}}{P_{\rm L}}$$

$$\lim_{n\to\infty}\left[\mathcal{L}_{\hat{X}^n_{\rm univ}}(P_{\mathbf{Z}},\Delta,\mathbf{Z})-\mu^{(n)}_{k_n}(P_{\mathbf{Z}},\Delta,\mathbf{Z})\right] = 0$$

$$P^*(e) = \frac{2\sqrt{\pi_0\pi_1(\bar{l}/\underline{l})^{m-1}}-1}{(\bar{l}/\underline{l})^{m-1}-1}$$

$$\max_{Ef(X)=\alpha} h(X) \simeq \max_\theta \frac{e^{\theta f(x)}}{\int e^{\theta f(x)}dx}\bigg|_{f(x)=\alpha}$$

$$\hat{S}_n \sim \frac{S_n^*(m-1)!(2\pi/n)^{(m-1)/2}}{|J_n|^{1/2}}$$

$$\lim_{n\to\infty}\sup_\theta\frac{1}{n}\log\frac{P_{\theta,Z}(\text{error}|\mathcal{C}_n)}{\bar{P}_{\theta,\rm ML}(\text{error})} = 0$$

$$h(X\,|\,X+Y) = h(Y\,|\,X+Y)$$

$$\frac{d^2}{dt^2}\exp\left(\frac{2h(\mathbf{X}+\sqrt{t}\mathbf{Z})}{n}\right)\le 0$$

$$R^* \le R \le R^*\left(2-\frac{M}{M-1}R^*\right)$$

$$\hat{\mathbf{b}}_{n+1} = \frac{\int \mathbf{b}S_n(\mathbf{b})\,d\mathbf{b}}{\int S_n(\mathbf{b})\,d\mathbf{b}}$$

$$\sum_{i\in S, j\in S^c} R^{(ij)} \le I(X^{(S)};Y^{(S^c)}|X^{(S^c)})$$

$$S_0 = \frac{2[\dot{C}(0)]^2}{-\ddot{C}(0)}$$

$$\sum_i p_i\,\varphi'(l_i) \le \varphi'\left(\varphi^{-1}\left(\sum_i p_i\,\varphi(l_i)\right)\right)$$

$$\tilde{C} = \bar{\alpha}C$$

$$\lim_{n\to\infty}\sup_{\theta\in\Theta}\frac{1}{n}\log\left(\frac{P_{\theta,u_n}(\text{error}|\mathcal{C}_n)}{\bar{P}_{\theta,\theta}(\text{error})}\right) = 0$$

$$h(X_1,X_2,\ldots,X_n) \le h(Z_1,Z_2,\ldots,Z_n) \le h(Z_1',Z_2',\ldots,Z_n')$$

$$H(Y_n\,|\,Y_{n-1},\ldots,Y_1) - H(Y_n\,|\,Y_{n-1},\ldots,Y_1,X_1) \longrightarrow 0$$

$$H(X^N) = (EN)H(X_1) + H(N|X^\infty)$$

$$\Sigma = \frac{C}{C_P\times C_F}$$

$$Q \ge \tfrac{1}{2}I(C;R|B)$$
$$Q+E \ge H(C|B)$$

$$r_{x_1,x_2,\cdots,x_h}(y) = \sum g_t p_{tx_t}(y)$$

$$\Pr\{N=n\} = \left(\frac{1}{2}\right)^n\binom{n-1}{d-1}$$

$$C_{\rm FB}(P) \lneq C(2P)$$

$$U \to X \to Y_1 \to Y_2$$

$$I(X;Y) = D(P_{Y|X}\,||\,Q\,|\,P_X) - D(P_Y\,||\,Q)$$

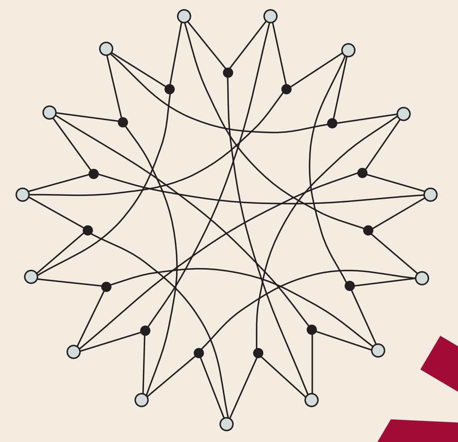
$$\log\frac{|K_X+K_Z^*|}{|K_Z^*|} \le \log\frac{|K_X^*+K_Z^*|}{|K_Z^*|} \le \log\frac{|K_X^*+K_Z|}{|K_Z|}$$

$$E\left[e^{Y-Y^*}\right]\le 1\quad\forall Y\in\mathcal{F} \iff E\left[Y-Y^*\right]\le 0\quad\forall Y\in\mathcal{F}$$

$$C = \sup_x\frac{D(P_{Y|X=x}||P_{Y|X=0})}{b[x]}$$

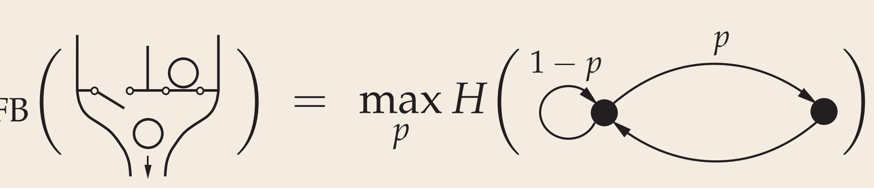
$$P_w^s = \frac{P_e(a_s,b_s)+P_w^{0s}P_w^{1s}}{2}$$

$$R_i \le \frac{1}{2}\ln\left(1+\frac{\alpha_iS}{N_i+\sum_{j<i}\alpha_jS}\right)$$



$$h(X+Y) \ge h(X'+Y')$$
$$V(A+B) \ge V(A'+B')$$

$$I(X;X+Z_G) \le I(X_G;X_G+Z_G) \le I(X_G;X_G+Z)$$



$$C_{\rm FB}(\cdot) = \max_p H(\cdot)$$

$$D \ge Q\frac{\gamma P+N}{\left(\sqrt{Q}+\sqrt{(1-\gamma)P}\right)^2+\gamma P+N}$$

$$R \le \frac{1}{2}\log\left(1+\frac{\gamma P}{N}\right)$$