Interference Management via Sliding-Window Superposition Coding

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Abstract—**The sliding-window superposition coding scheme achieves the performance of simultaneous decoding with point-to-point channel codes and low-complexity decoding. This paper provides a case study of how this conceptual coding scheme can be transformed to a practical coding technique for two-user Gaussian interference channels. Simulation results demonstrate that sliding-window superposition coding can sometimes double the performance of the conventional method of treating interference as noise, still using the standard LTE turbo codes.**

I. Introduction

For high data rates and massive connectivity, the next-generation cellular networks are expected to deploy many small base stations. While such dense deployment provides the benefit of bringing radio closer to end users, it also increases the amount of interference from neighboring cells. Consequently, smart management of interference would become one of the key enabling technologies for high-spectral-efficiency, low-power, broad-coverage wireless communication.

Over the past decades, several techniques at different protocol layers have been proposed to mitigate adverse effects of interference in wireless networks; see, for example, [1]–[3]. One important conceptual technique at the physical layer is simultaneous decoding [4], [5], whereby each receiver decodes for the desired signal as well as part or whole of interference. When interference is strong [6], [7], this simultaneous decoding technique achieves the optimal performance for the two-user Gaussian interference channel using good point-to-point codes. Moreover, it achieves the optimal maximum likelihood decoding performance in general, when the encoders are restricted to point-to-point random code ensembles [8], [9]. The celebrated Han-Kobayashi coding scheme [10], which achieves the best known performance for general two-user interference channels, also uses simultaneous decoding as a crucial component. As a main drawback, however, each receiver in simultaneous decoding has to employ some form of multiuser sequence detection, which usually requires high computational complexity to implement. This issue has been tackled lately by a few approaches [11], [12] based on emerging spatially coupled [13] and polar [14] codes, but these solutions require the development of new families of codes (instead of using conventional point-to-point channel codes such as LDPC codes [15] and turbo codes [16]) and involve very long block lengths.

This work was supported in part by the National Science Foundation under Grant CCF-1320895.

For this reason, most existing communications systems, which use conventional point-to-point channel codes, treat interference as noise. While this simple scheme can achieve good performance with low computational complexity when interference is weak [17]–[19], the performance degrades as interference becomes stronger, which is often the case for dense wireless networks. In particular, in the high signal-to-noise ratio/interference-to-noise ratio limit, the performance of treating interference as noise has an unbounded gap from that of simultaneous decoding.

Recently, the sliding-window superposition coding scheme was proposed [20] that achieves the theoretical performance of simultaneous decoding with point-to-point channel codes and low-complexity decoding. This scheme is built on basic components of network information theory, combining the ideas of block Markov coding and sliding-window decoding (commonly used for multihop relaying and feedback communication, but not for single-hop communication) and superposition coding and successive cancellation decoding (allowing low-complexity decoding with point-to-point codes).

In this paper, we investigate the performance of sliding-window superposition coding (SWSC) for the two-user Gaussian interference channel and demonstrate that SWSC provides a feasible solution to achieve the performance of simultaneous decoding with existing point-to-point codes. We first evaluate the theoretical performance of SWSC under modulation constraints and compare it with the performance of treating interference as noise and simultaneous decoding. As is fully described in Section III, SWSC tracks the performance of simultaneous decoding when interference is moderate to strong. To further test the feasibility of SWSC, we then translate the conceptual coding scheme behind the theoretical performance to a practical implementation based on actual turbo codes used in the LTE standard [21]. In Section IV, we show that our implementation achieves the performance guaranteed by the theory, even when the block length is relatively short (2048). In particular, our implementation outperforms conventional systems that treat interference as noise when interference is moderate to strong.

In the next section, we begin our discussion by reviewing the basic operations of SWSC [20] for the special case of the two-user Gaussian interference channel.

978-1-4799-7470-2/14/$31.00

II. Sliding-Window Superposition Coding for the Gaussian Interference Channel

The two-user Gaussian interference channel is defined as

$$\begin{aligned} Y_1 &= g_{11}X_1 + g_{12}X_2 + Z_1, \\ Y_2 &= g_{21}X_1 + g_{22}X_2 + Z_2. \end{aligned} \tag{1}$$

Here, $X_i \in \mathcal{X}^n$, $i = 1, 2$, is a transmitted signal from sender i with average power constraint P_i, where n is the block length and $Y_i \in \mathbb{R}^n$ is a received signal at receiver i, and $Z_i \in \mathbb{R}^n \sim \mathrm{N}(0, 1)$, $i = 1, 2$, are noise components. We assume that each receiver i knows local channel gain coefficients $g_{ij} \in \mathbb{R}$, $j = 1, 2$, from both senders, which are held fixed during the communication. The block diagram of this channel is shown in Fig. 1.

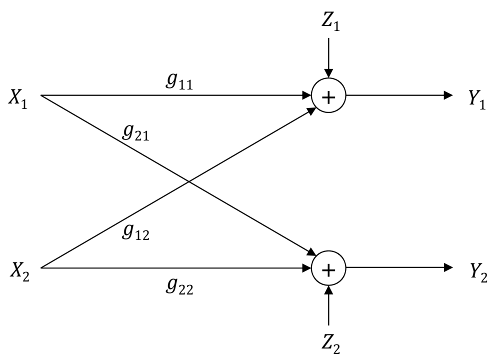


Fig. 1. The two-user Gaussian interference channel.

Sliding-window superposition coding (SWSC) [20] is based on several basic building blocks in network information theory such as superposition coding [22], block Markov coding [23], [24], successive cancellation decoding [22], [25], [26], and sliding-window decoding [27]–[29]. Sender i encodes its messages by using superposition coding with multiple superimposed layers and block Markov coding throughout multiple blocks. Receiver i performs successive cancellation decoding of all superimposed layers from sender i and some superimposed layers from the other sender within a window length according to a predetermined decoding order and slides the decoding window until it reaches the end of blocks.

We now elaborate the encoding/decoding process of the specific version of SWSC considered in this paper.

For block $j = 1, \ldots, b$, let $m_i(j) \in \{1, 2, \ldots, 2^{nr_i}\}$ be the message to be communicated from sender i to receiver i. Similarly, let $X_i(j)$, $Y_i(j)$, and $Z_i(j)$ be the channel input, output, and noise for sender/receiver i in block j.

The original SWSC allows for full flexibility in the number of superimposed layers, the number and structure of auxiliary random variables for superposition coding, and the decoding order. Here, we limit our attention to two layers of BPSK signals that form a 4-PAM signal by superposition and a fixed decoding order. In particular,

$$\begin{aligned} X_1(j) &= \sqrt{P_1}\sqrt{\alpha}U(j) + \sqrt{P_1}\sqrt{1-\alpha}V(j), \\ X_2(j) &= \sqrt{P_2}W(j), \end{aligned} \tag{2}$$

where $U(j)$, $V(j)$, and $W(j) \in \{-1, +1\}^n$ are BPSK signals. Fig. 2 represents the superposition coding with U and V for X_1.

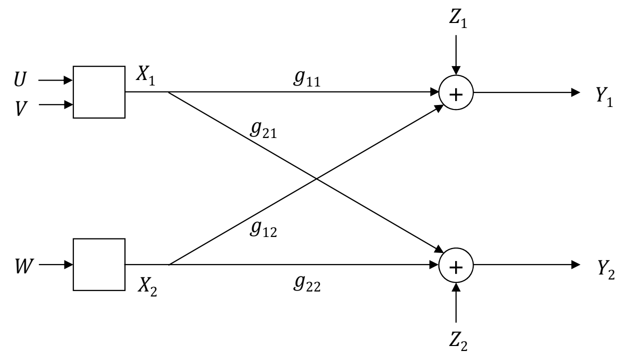


Fig. 2. Superposition coding with virtual input signals U, V, and W.

The encoding and decoding operations are depicted in Fig. 3. The signal $U(j)$ carries the message $m_1(j-1)$ from the previous block, and $V(j)$ and $W(j)$ carry $m_1(j)$ and $m_2(j)$, respectively, from the current block. By convention, we set $m_1(0) = m_1(b) = 1$. The parameter α determines the ratio of powers split into $U(j)$ and $V(j)$. Throughout this paper, $\alpha = 0.8$, which makes $X_1 \in \{-3\sqrt{P_1}/\sqrt{5}, -\sqrt{P_1}/\sqrt{5}, +\sqrt{P_1}/\sqrt{5}, +3\sqrt{P_1}/\sqrt{5}\}$ a *uniformly-spaced* 4-PAM signal.

The corresponding channel outputs are

$$\begin{aligned} Y_1(j) &= g_{11}\sqrt{P_1}\sqrt{\alpha}U(j) + g_{11}\sqrt{P_1}\sqrt{1-\alpha}V(j) \\ &\quad + g_{12}\sqrt{P_2}W(j) + Z_1(j), \\ Y_2(j) &= g_{21}\sqrt{P_1}\sqrt{\alpha}U(j) + g_{21}\sqrt{P_1}\sqrt{1-\alpha}V(j) \\ &\quad + g_{22}\sqrt{P_2}W(j) + Z_2(j). \end{aligned}$$

At the end of block $j+1$, receiver 1 first decodes $Y_1(j)$ and $Y_1(j+1)$ to recover $m_1(j)$ carried by $V(j)$ and $U(j+1)$. Note that $U(j)$ and $W(j)$ are already known from the previous decoding window and thus the effective channel output from $Y_1(j)$ is $g_{11}\sqrt{P_1}\sqrt{1-\alpha}V(j) + Z_1(j)$. This decoding step is successful if

$$r_1 \le I(U; Y_1) + I(V; Y_1|U, W). \tag{3}$$

Receiver 1 then decodes $Y_1(j+1)$ to recover $m_2(j+1)$ carried by $W(j+1)$, where $U(j+1)$ is known from the first step and $V(j+1)$ is interference. This decoding step is successful if

$$r_2 \le I(W; Y_1|U). \tag{4}$$

At the end of block $j+1$, receiver 2 first decodes $Y_2(j)$ and $Y_2(j+1)$ to recover $m_1(j)$ carried by $V(j)$ and $U(j+1)$, where $U(j)$ is known from the previous decoding window and $V(j)$ is interference. Receiver 2 then decodes $Y_2(j)$ to recover $m_2(j)$ carried by $W(j)$, where $U(j)$ and $V(j)$ are already known. These decoding steps are successful if

$$r_1 \le I(U, V; Y_2), \tag{5}$$

$$r_2 \le I(W; Y_2|U, V). \tag{6}$$

block	1	2	3	4	5	6	7
U		$m_1(1)$	$m_1(2)$	$m_1(3)$	$m_1(4)$	$m_1(5)$	$m_1(6)$
V	$m_1(1)$	$m_1(2)$	$m_1(3)$	$m_1(4)$	$m_1(5)$	$m_1(6)$	
W	$m_2(1)$	$m_2(2)$	$m_2(3)$	$m_2(4)$	$m_2(5)$	$m_2(6)$	$m_2(7)$
		decoding at receiver 1			decoding at receiver 2		

Fig. 3. The encoding and decoding operations for $b = 7$ blocks. The message $m_1(2)$ is carried by signals $V(2)$ and $U(3)$ (shaded in blue), while the message $m_2(5)$ is carried by $W(5)$ (shaded in red). The sliding-window decoding of $m_1(2)$ at receiver 1 is based on its received signals $Y_1(2)$ and $Y_1(3)$ over two blocks. Receiver 1 first recovers $m_1(2)$ (equivalently, $V(2)$ and $U(3)$) and then recovers $m_2(3)$ (equivalently, $W(2)$). The signals $U(2)$ and $W(2)$ are already known from the previous decoding window (shaded in gray). Receiver 2 operates slightly differently by recovering first $m_1(5)$ from and then $m_2(5)$ based on two blocks $Y_1(5)$ and $Y_1(6)$.

At the end of the last block $j = b$, receiver 2 addtionally decodes $Y_2(b)$ to recover $m_2(b)$ carried by $W(b)$, which is again successful if (6) holds.

Since $m_1(1), \ldots, m_1(b-1)$ and $m_2(1), \ldots, m_2(b)$ are sent over b blocks, the actual rate for sender/receiver 1 is $R_1 = r_1(b-1)/b$ and the actual rate for sender/receiver 2 is $R_2 = r_2$. Combining these results (4)–(6), we can asymptotically achieve the following rate region with SWSC:

$$\begin{aligned} R_1 &\le \min\{I(U;Y_1) + I(V;Y_1|U,W), I(U,V;Y_2)\}, \\ R_2 &\le \min\{I(W;Y_1|U), I(W;Y_2|U,V)\}, \end{aligned} \tag{7}$$

where U, V, and W are independent Unif$\{-1,+1\}$ random variables.

III. Theoretical Performance Comparison

In this section, we consider treating interference as noise and simultaneous decoding for the channel model in (1) and compare the theoretical performance of SWSC to them.

A. Treating Interference as Noise

The achievable rate region of treating interference as noise is characterized by

$$\begin{aligned} R_1 &\le I(X_1;Y_1), \\ R_2 &\le I(X_2;Y_2), \end{aligned} \tag{8}$$

where X_1 is Unif$\{-3\sqrt{P_1}/\sqrt{5}, -\sqrt{P_1}/\sqrt{5}, +\sqrt{P_1}/\sqrt{5}, +3\sqrt{P_1}/\sqrt{5}\}$ and X_2 is Unif$\{-\sqrt{P_2}, +\sqrt{P_2}\}$ as in (2). Note that the receivers here use the constellation (modulation) information of interference instead of the simple signal-to-interference-noise ratio (SINR) metric, but they do not decode for the interference codewords.

B. Simultaneous Nonunique Decoding

In simultaneous decoding, each receiver recovers codewords from both senders. Here we consider a variant called *simultaneous nonunique decoding*, which provides an improved performance by disregarding the uniqueness of the interference codeword [30]. The achievable rate region of simultaneous nonunique decoding is characterized by

$$\begin{aligned} R_1 &\le I(X_1;Y_1|X_2), \\ R_2 &\le I(X_2;Y_2|X_1), \\ R_1 + R_2 &\le \min\{I(X_1,X_2;Y_1), I(X_1,X_2;Y_2)\}, \end{aligned} \tag{9}$$

where X_1 and X_2 are again given as in (2). Note that simultaneous nonunique decoding achieves the capacity region when interference is strong, that is, $g_{21}^2 \ge g_{11}^2$ and $g_{12}^2 \ge g_{22}^2$.

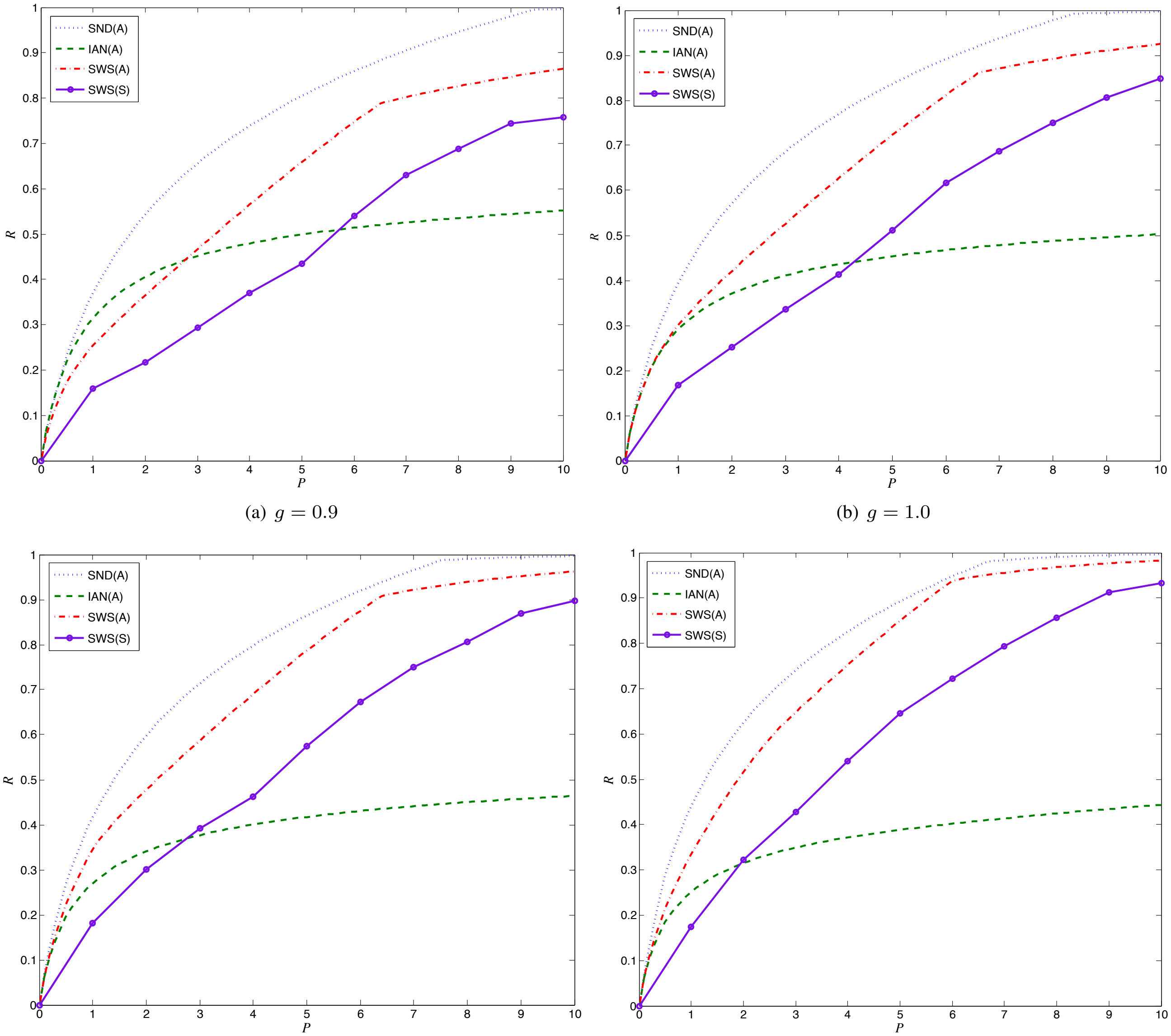
C. Comparison with SWSC

We demonstrate the theoretical performance of sliding-window superposition coding (SWS(A)) compared to the theoretical performance for simultaneous nonunique decoding (SND(A)) and treating interference as noise (IAN(A)). In the original SWSC scheme, the auxiliary signals U, V, and W as well as the superposition mapping $x_1(u,v)$ can be chosen optimally, which guarantees that SWS(A) is identical to SND(A). In our setting, however, we have restricted the auxiliary signals to be BPSK so that X_1 is uniformly-spaced 4-PAM. Therefore, it is *a priori* unclear whether SWS(A) would be close to SND(A).

For simplicity, assume the symmetric rate, power, and channel gains, that is, $R_1 = R_2 = R$, $P_1 = P_2 = P$, $g_{11} = g_{22} = 1$, and $g_{12} = g_{21} = g$. We control signal-to-noise ratio (SNR) and interference-to-noise ratio (INR) by varying transmit power P and find the minimum power P that achieves the given rate R for SND(A), IAN(A), and SWS(A). The plots of the minimum symmetric transmit power P vs. the achievable symmetric rate R are shown in Fig. 4 for $g = 0.9,\ 1.0,\ 1.1,\ 1.2$. Note that the gap between SWS(A) and SND(A) is due to the suboptimal choice of U and V under our modulation constraints. Nonetheless, SWS(A) approaches SND(A) and significantly outperforms IAN(A) in high SNR.

IV. Implementation with LTE Turbo Codes

To implement SWSC with point-to-point channel codes, we use a binary linear code of length $2n$ and rate $r_1/2$ for $[V(j)|U(j+1)]$ as $U(j+1)$ and $V(j)$ carry $m_1(j)$ in common. Similarly, we use a binary linear code of length n and rate r_2 for $W(j)$ to carry $m_2(j)$. We adopt the turbo codes used in the LTE standard [21], which allow flexibility in code rate and block length. In particular, we start with the rate $1/3$ mother code and adjust the rates and lengths according to the rate matching algorithm in the standard. Note that for $r_1 < 2/3$, some code bits are repeated and for $r_1 > 2/3$, some code bits are punctured. To evaluate the performance of SWSC, the block length n and the number of blocks b are set to 2048



(a) $g = 0.9$ (b) $g = 1.0$ (c) $g = 1.1$ (d) $g = 1.2$

Fig. 4. The transmit power P vs. achievable symmetric rate R for the symmetric Gaussian interference channel.

and 20,[1] respectively. We use the LOG-MAP algorithm for the turbo decoding with the maximum number of iterations set to 8 for each stage of decoding. We assume that a rate pair (R_1, R_2) is achieved for given P_i and g_{ij} if the resulting bit-error rate (BER) is below 10^{-3} over 1000 independent sets of simulations.

We first consider the symmetric case studied in the previous section. Our simulation results are overlaid in Fig. 4 along with the theoretical performance curves. It can be checked that the performance of our implementation, SWS(S), tracks the theoretical performance of SWS(A), confirming the feasibility of sliding-window superposition coding. Note that SWS(S) outperforms IAN(A) in high SNR. Recall that the latter is the theoretical performance bound of treating interference as noise, whose actual performance (under a fair comparison) would be even worse.

As another feasibility test, we consider the Gaussian fading interference channel, where g_{ij} are i.i.d. $\sim \mathrm{N}(0, 1)$. We independently generate 25 sets of channel gain coefficients, in order to evaluate the performance of SWSC under various channel conditions. We calculate the average minimum power P_{avg} over the 25 channel realizations for $R = 0.3,\ 0.4,\ 0.5,\ 0.6$. As shown in Fig. 5, SWS(A) is very close to SND(A), which is tracked by the actual implementation SWS(S). Note that SWS(S) is consistently better than IAN(A), with the gap becoming larger in high rate/high SNR regime.

[1]It should be stressed that b is the total number of blocks, not the size of the decoding window (which is 2). Every message is recovered with one-block delay. While a larger b reduces the rate penalty of $1/b$, it also incurs error propagation over multiple blocks, both of which were properly taken into account in our rate and BER calculation.

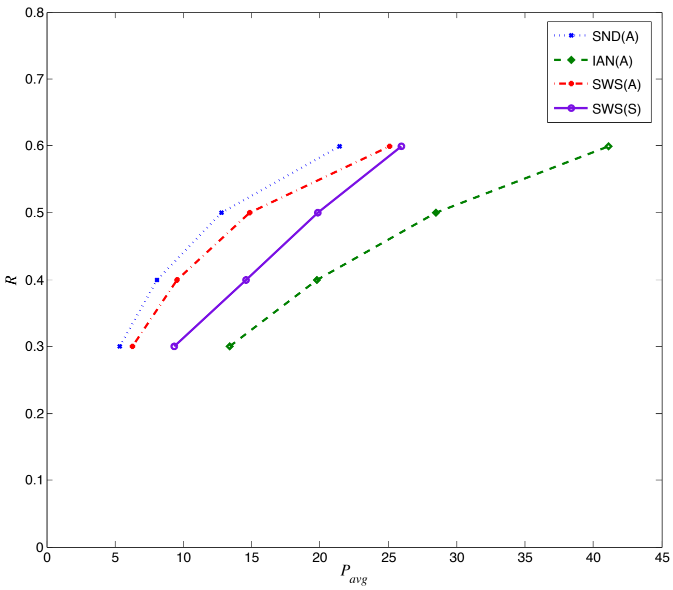


Fig. 5. The average transmit power vs. achievable symmetric rate for the Gaussian interference channel with random coefficients.

V. Concluding Remarks

While there should be more extensive studies on its feasibility, the results in this paper indicate that the sliding-window superposition coding (SWSC) scheme has some potential as a practical channel coding technique for interference management. We remark on two directions in improving the current implementation. First, the decoding orders at the receivers can be further optimized; for example, SWSC can always achieve the performance of treating interference as noise under certain decoding orders. Second, the structure of the superposition mapping can be further optimized, especially, by the power ratio control ($\alpha \neq 0.8$).

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