Rate vs. Distortion Trade-off for Channels with State Information[1]

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Abstract — **We consider a channel where the sender has access to channel state information and wishes to send both the pure information and a description of the channel state to the receiver. An achievable trade-off region between pure information rate and state estimation error is proposed for a discrete memoryless channel with an arbitrary state distortion measure.**

I. Introduction

Consider communication with state information as depicted in Figure 1. In addition to transmitting the pure information, the sender also wishes to transmit a description of the channel state to the receiver. More formally, the transmitter wishes to send pure information $W \in \{1, 2, \cdots, 2^{nR}\}$ and the description $\hat{S}^n$ of the state S^n in n uses of a memoryless channel $p(y|x, s)$. The state sequence $\{S_1, S_2, \cdots, S_n\}$ is assumed to be independent and identically distributed according to $p(s)$. Based on the state S^n and pure information index W, the transmitter transmits $X^n(W, S^n)$, $W \in \{1, 2, \cdots, 2^{nR}\}$. The rate of this code is R. Upon receiving $Y^n \sim \prod_{i=1}^n p(y_i|x_i, s_i)$, the receiver guesses $\hat{W}(Y^n)$ and forms an estimate of the state $\hat{S}^n(Y^n)$. The probability of decoding error $P_e^{(n)}$ and the state estimation error D are given by $P_e^{(n)} = \frac{1}{2^{nR}} \sum_{i=1}^{2^{nR}} Pr\{\hat{W} \neq i | W = i\}$ and $D = Ed(S^n, \hat{S}^n) = \frac{1}{n} \sum_{i=1}^n Ed(S_i, \hat{S}_i)$, respectively, where $d(s, \hat{s})$ is a distortion measure between S and $\hat{S}$. We say (R, D) is *achievable* if there exist codes $X^n(W, S^n)$, $W \in \{1, 2 \cdots, 2^{nR}\}$, and $\hat{S}^n(Y^n)$, with $P_e^{(n)} \to 0$ and $\overline{\lim} Ed(S^n, \hat{S}^n) \leq D$.

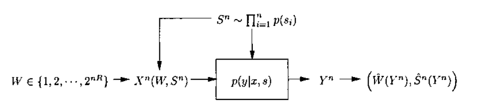

Figure 1: Channel model: $Y^n \sim \prod_{i=1}^n p(y_i|x_i, s_i)$, with state $S^n \sim \prod_{i=1}^n p(s_i)$ known at transmitter.

In [3], the optimal trade-off region between pure information rate R and mean-squared estimation error $D = Ed(S - \hat{S})^2$ was obtained for an additive Gaussian channel, $Y^n = X^n(W, S^n) + S^n + Z^n$, with state S^n known at the transmitter. In this work, we prove an achievable trade-off region for a general discrete memoryless channel with an arbitrary distortion measure.

II. Main results

Theorem 1 *For a discrete memoryless channel $p(y|x, s)$ with state $S^n \sim \prod_{i=1}^n p(s_i)$ noncausally known at the transmitter and a state distortion measure $d(s, \hat{s})$, an achievable (R, D) trade-off region is the closure of the convex hull of the set of all (R, D) pairs, $R \geq 0$, satisfying*

$$\begin{aligned} R &\leq I(U;Y) - I(U;S) - I(V;S|U) + I(V;Y|U) \\ D &\geq Ed(S, \hat{S}), \end{aligned}$$

for some distribution $p(u|s)p(v|u, s)p(x|u, s)p(\hat{s}|v, u, y)$.

A sketch of the proof is as follows. Fix a distribution $p(u|s)p(v|u, s)p(x|u, s)p(\hat{s}|v, u, y)$. The maximum information rate that can be transmitted reliably across the channel is $I(U;Y) - I(U;S)$, as given in [1]. These available bits are allocated between pure information transmission and state description. The information-bearing signal U is chosen by the transmitter so that it facilitates the transmission and is distinguishable at the receiver. Because of their inherent dependency on the state S, the received signal Y and the decoded signal U can also be used to facilitate the state estimation. As a result, the minimum rate needed in describing the state S with distortion D corresponds to the minimum rate needed when side information U is available at the transmitter and (U, Y) is available at the receiver, which is given by $I(V; U, S) - I(V; U, Y) = I(V; S|U) - I(V; Y|U)$ bits [2], where V is an auxiliary random variable used to facilitate the state estimation at the receiver. With high probability the resulting distortion will be $D = Ed(S, \hat{S})$. The remaining $I(U;Y) - I(U;S) - I(V;S|U) + I(V;Y|U)$ bits can then be used to transmit pure information.

The proposed trade-off region can be shown to be optimal for certain classes of channels and distortion measures, including the additive Gaussian channel with a mean-squared distortion measure in [3].

III. Concluding remarks

Pure information transmission and state description transmission represent two conflicting goals. Pure information transmission usually corrupts the state of the channel—rendering it more difficult for the receiver to ascertain the channel state. State information transmission, on the other hand, necessarily uses up the resources that can otherwise be used in transmitting pure information. A characterization of the optimal trade-off region reveals the interplay between the two competing objectives.

References

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