A Game-Theoretic Approach to Coding for Information Networks

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***Abstract*—Since its inception, network coding literature has, for the most part, assumed cooperation among users. In unicast applications where users have no inherent interest in providing (or concealing) their information to (or from) any destinations except for their unique one, this assumption must be reconsidered. In this paper, we examine the impact of selfish users on coding strategies by formulating network coding games, in which users strategies are their encoding/decoding schemes (including encoding functions, block length, rate, etc). Through the use of examples, we show that the rational outcomes of such network coding games are dependent on the particular network coding scheme implemented at intermediate nodes in the network. More specifically, we construct examples that show how careful construction of network coding schemes at intermediate nodes in the network can guarantee that efficient coding solutions will emerge as a rational outcome of the game, even when users are allowed complete freedom in choosing their coding schemes.**

I. Introduction

Recently, there has been a surge of research in the field of network information theory. Fueled by advances in physical layer technology, researchers have begun to construct sophisticated coding techniques and communication schemes for increased reliability and communication rates in multi-user networks. Although elegant, efficient, and reliable, communication schemes based on network information theoretic results may require cooperation among users in order to achieve the desired performance. In the presence of selfish users, the assumption that users will voluntarily cooperate by employing the desired communication scheme cannot easily be justified.

One area in which this becomes a critical issue is in the field of network coding. Since its inception in the seminal work of Ahlswede, et. al, [1], network coding literature has, for the most part, assumed cooperation among users. In unicast applications where users have no inherent interest in providing (or concealing) their information to (or from) any destinations except for their unique one, this assumption must be re-examined.

Consider, for example, the classical *butterfly* network that was introduced in [1] and is shown in Figure 1(a). In the classical butterfly network (and its generalization, degree-2 three layer networks [2]) with two unicast flows, the use and utility of side links $(S(1), D_1)$ and $(S(2), D_2)$ is limited to the possible carrying of so-called *side* or redundant information that can only be used in decoding. As a result, in the classical butterfly network there is little contention

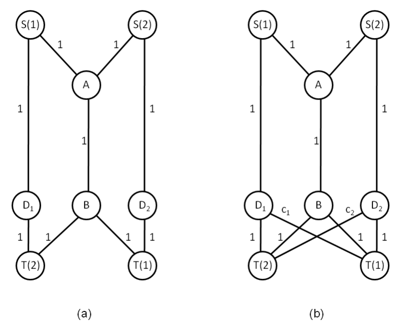

Fig. 1. (a) The Classical Butterfly Network. (b) The Extended Butterfly Network

over the use of links $(S(1), D_1)$ and $(S(2), D_2)$. In other words, even if the desired coding scheme requires both users to completely fill side links $(S(1), D_1)$ and $(S(2), D_2)$ with redundant information for use in decoding, there is no disincentive for users to participate since these links are otherwise unusable.

Not all networks are as robust to non-cooperation, however. Consider what happens if a user is faced with the choice of using its resources either for transfer of new information to its own destination, or for cooperation in the "desired" network coding scheme. This is exactly the situation illustrated in Figure 1(b), which shows a simple generalization of the classical butterfly network consisting of the same two unicast sessions over a network. In this network, however, the link $(S(1), D_1)$ $[(S(2), D_2)]$ can be used *either* to carry part of the unicast flow from S(1) to T(1) [from S(2) to T(2)] *or* to provide side information from S(1) to T(2) [from S(2) to T(1)]. At first glance, it is unclear how a user will behave in this situation. It turns out that whether a rational source $S(i)$ finds it advantageous to use link $(S(i), D_i)$ for transfer of fresh or side information is determined by the network coding scheme implemented at intermediate nodes. Thus, while designing network codes that ensure users have an incentive to participate might have been considered a toy problem in an information-theoretic context, it becomes a very real concern when considering practical implementations of network coding in unicast networks.

In this paper, we examine the impact of the network coding scheme implemented by the network on the behavior of selfish, rational users. Specifically, we show that careful selection of network codes can be used to ensure that users have incentive to participate in the desired cooperative communication schemes, even when they are given complete autonomy in choosing their coding schemes.

The remainder of the paper is organized as follows. Section II describes the problem formulation in terms of network setting and user coding schemes. Section III gives a complete characterization of the individually-rational operating points of our network coding game, and examines what happens when the operating point is chosen in a distributed manner. Section IV presents a modified network coding scheme, and shows that this scheme leads to an optimal operating point that can be achieved in dominant strategies. Finally, Section V presents our conclusions and areas of future work.

II. Problem Formulation

In this paper, we work with the extended butterfly network shown in Figure 1(b). Each user has a message, given by

$$W_i = [b_{i,1}, \ldots, b_{i,B_i}]$$

where $b_{i,j}$ are information bits, and B_i is the total number of information bits sent by user i. The message W_i is then encoded over block length N_i and sent from $S(i)$ to $T(i)$ over the network. In order to allow users complete freedom in choosing their coding strategy, users can generate two sets of codewords: one set X_i^M that is routed over the middle portion of the network (i.e. through node A), and one set X_i^S that is routed over the side portion of the network (i.e. through node D_i. The complete set of codewords is as follows.

X_i^{MUS}:	The unicast codeword sent from $S(i)$ to $T(i)$ through node A.
X_i^{MUO}:	The unicast codeword sent from $S(i)$ to $T(-i)$ through node A.
X_i^{MB}:	The broadcast codeword that is sent from $S(i)$ to $T(i)$ and $T(-i)$ through node A.
X_i^{SUS}:	The unicast codeword sent from $S(i)$ to $T(i)$ through node D_i.
X_i^{SUO}:	The unicast codeword sent from $S(i)$ to $T(-i)$ through node D_i.
X_i^{SB}:	The broadcast codeword that is sent from $S(i)$ to $T(i)$ and $T(-i)$ through node D_i.

By allowing users to generate any combination of these six codewords, we allow users complete freedom in determining not only how to encode their message, but also how to route these codewords through the network. Thus, user i's encoding function is given by

$$\begin{aligned} f_i(W_i) &= [X_i^M, X_i^S] \\ &= [X_i^{MUS}, X_i^{MUO}, X_i^{MB}, X_i^{SUS}, X_i^{SUO}, X_i^{SB}] \end{aligned}$$

where each codeword is a series of binary digits (bits) whose length is denoted by $|\cdot|$. The encoding functions must satisfy traditional link capacity constraints, given by:

$$\frac{1}{N_i}|X_i^M| \leq 1 \quad (1)$$

$$\frac{1}{N_i}|X_i^S| \leq 1 \quad (2)$$

$$\frac{1}{N_i}|X_i^{SUS}| + \frac{1}{N_i}|X_i^{SB}| \leq c_i \quad (3)$$

Finally, we need to specify the network coding scheme used by the network. Since users are allowed to choose any block length, we define super-blocks whose length is given by

$$\begin{aligned} N &= LCM(N_1, N_2) \\ &= k_1 N_1 \\ &= k_2 N_2 \end{aligned}$$

During each super-block, Node A performs XOR coding of bits from the two sources to generate a network-coded symbol, $Z = h(X_1^M, X_2^M) = [X_{1,1}^M \bigoplus X_{2,1}^M, \ldots, X_{1,N}^M \bigoplus X_{2,N}^M]$. All other nodes simply forward bits to the appropriate destination.

III. Non-Cooperative Network Coding Game

In order to examine the behavior of the unicast flows, we formulate codebook selection (as described in the previous section) as a non-cooperative game where the players are the unicast flows. Each user is a rational but selfish player, interested in maximizing its own utility without regard to its impact on other users or the system as a whole. The network is a passive entity whose coding strategy (as described in the previous section) is fixed and known to users apriori.

Since we are modeling this as a one-shot non-cooperative game, users select their strategies (codebooks) simultaneously. In other words, while users can assume that they will know the codebook of the other player at their receiver, they do not have this information at the transmitter when designing their codebook.

Before presenting a formal description of the game, we must first examine what we mean when we say that a strategy is *individually rational*. Intuitively, individual rationality means that users are playing the strategy that will give them the highest utility. There are, however, varying degrees to which a strategy can be individually rational. To that end, we introduce the following definitions, each of which represent a type of individual rationality.

Definition 1: Suppose user 1 chooses strategy σ_1. The strategy σ_2 is user 2's *best response* to strategy σ_1 if it satisfies $\sigma_2 = \arg\max_{\hat{\sigma}_2 \in \Sigma_2} U_2(\sigma_1, \hat{\sigma}_2)$.

Definition 2: A pair of strategies (σ_1, σ_2) is a *Nash equilibrium* if $\sigma_1 = \arg\max_{\hat{\sigma}_1 \in \Sigma_1} U_1(\hat{\sigma}_1, \sigma_2)$ and $\sigma_2 = \arg\max_{\hat{\sigma}_2 \in \Sigma_2} U_2(\sigma_1, \hat{\sigma}_2)$.
In other words, σ_1 and σ_2 are a Nash equilibrium if they are best responses to one another.

Definition 3: A strategy σ_2 is a *dominant strategy* for user 2 if it satisfies $\sigma_1 = \arg\max_{\hat{\sigma}_1 \in \Sigma_1} U_1(\hat{\sigma}_1, \sigma_2)\ \forall \sigma_2 \in \Sigma_2$.

These definitions can be found, for example, in [3], [4].

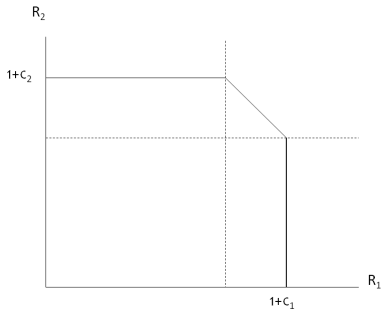

Fig. 2. Capacity Region of the Extended Butterfly Network

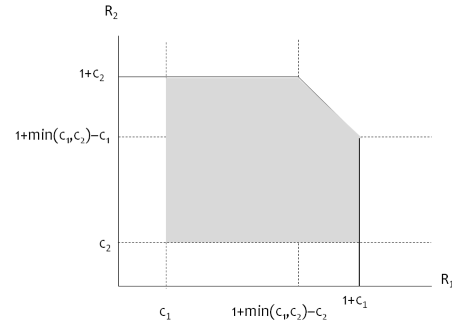

Fig. 3. Subset of the Capacity Region Achievable as a Nash Equilibrium

A. Game Formulation

Formally, we denote the non-cooperative network coding game by $\mathcal{F} = [\mathcal{N}, \{\sigma_i\}, \{U_i(\cdot)\}]$. Here, $\mathcal{N} = \{1, 2\}$ is the set of players, where $S(i)$ and $T(i)$ are the source and destination nodes for unicast flows $i = 1, 2$. The strategy space of player i is denoted by Σ_i, and consists of all possible strategies $\sigma_i = \{N_i, B_i, f_i(\cdot)\}$ where $N_i \geq 0$, $B_i \geq 0$, and $f_i(\cdot)$ satisfy the link capacity constraints given by (1)-(3). Finally, we model the utility of user i as $U_i(\sigma_i, \sigma_{-i}) = \sum_{j=1}^{B_i} 1(b_{i,j})$, where

$$1(b_{i,j}) = \begin{cases} 1 & \text{if } b_{i,j} \text{ is correctly decoded} \\ 0 & \text{else} \end{cases}$$

is an indicator function for information bit $b_{i,j}$ being correctly decoded at its own receiver.

It is worth noting that since we gain utility for an information bit only if it is correctly decoded at the receiver, we are looking at zero-error capacity, in the spirit of [5]. In other words, we are not allowing for decreasing probability of error as block length increases; rather, we consider cases in which the probability of error is zero.

B. Nash Equilibrium Characterization

In order to determine what (if any) pairs of strategies are both individually-rational and capacity-achieving, we must first determine the capacity region of the network. It has been shown in [6] and [7] that the capacity region of this network is given by:

$$R_1 \leq 1 + c_1 \tag{4}$$

$$R_2 \leq 1 + c_2 \tag{5}$$

$$R_1 + R_2 \leq 2 + \min(c_1, c_2) \tag{6}$$

Figure 2 shows this capacity region. We denote by Δ the set of rates that satisfy (4)-(6).

It turns out that not only is it possible to find a pair of strategies that are a Nash equilibrium, but that a large subset of the capacity region can be achieved as a Nash equilibrium. To that end, we completely characterize the operating points that can be achieved as a Nash equilibrium in the following theorem.

Theorem 1: Let Δ be the capacity of the network, and let $\mathcal{NE} = \{(R_1, R_2) : R_1 \geq c_1,\ R_2 \geq c_2$. Then all points in the region $\cdot_{\mathcal{NE}} = \Delta \cap \mathcal{NE}$ can be achieved as Nash equilibria.

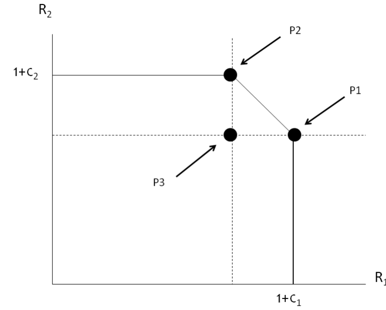

Fig. 4. Particular Operating Points in the Capacity Region

Proof: See Appendix I. ■

This region is depicted in Figure 3. Note that this region is similar to the one described in [8] for a deterministic model of a Gaussian interference channel.

C. Distributed Selection of Operating Points

The benefit of using game-theoretic analysis is that it captures the behavior of rational, selfish users who are given autonomy in picking their strategies. Unfortunately, while game-theoretic analysis of a simultaneous move game can identify individually rational operating points, it typically gives little or no insight as to how to achieve such operating points. In other words, how do users choose Nash equilibrium strategies in a distributed manner?

This question becomes critical in the case of the non-cooperative network coding game described above. Since there are many capacity-achieving (and non-capacity achieving) Nash equilibrium, it is unclear where users will choose to operate. Furthermore, individual users clearly have preferences for which Nash equilibrium they choose to operate at. Consider the operating points shown in Figure 4. User 1 prefers to operate at point P1, but user 2 prefers to operate a point P2. If both users aim for their respective desired points, the network actually ends up operating at point P3 - a point which is non-capacity achieving and in which both users are worse off than had they cooperatively chosen a different operating point. In economics, this is known as a *tragedy of commons*, in which the selfish interests of the users exhaust the resources of the network [3], [4].

In the next section, we introduce a modification to the non-cooperative network coding game that results in a dominant strategy equilibrium. In other words, users always know what their best response strategy is - regardless of what strategy the other user chooses.

IV. Modified Network Coding Schemes

We have seen that when the network performs a "blind" network coding scheme in which it simply XOR's all bits at Node A, there are many operating points that can be achieved as Nash equilibrium. It is difficult, however, to find even a single operating point that can be achieved in dominant strategies. This is because each user faces an interference channel, where the interference is a function not only of the network-coding scheme employed by the network, but by the codebook selection of the other user. In short, players are faced with the difficult problem of designing their codebooks for an unknown channel.

To remedy this, we propose a modified network coding scheme in which the network does something more than simply XOR bits. Instead, it combines a bidding process with a reservation scheme in order to explicitly shape the interference seen by the users. Intuitively, this game consists of three stages:

1. Users submit bids for how much information they are willing to network code (i.e. how much information they are willing to send to the other user for use in decoding).
2. The network examines the bids, and determines how many bits will be network coded (XOR'd) by choosing the minimum of the two bids, normalized to the super-block length. It also determines how many bits from each user will be routed (without being coded by the network) in the remainder of the super-block.
3. Users receive the parameters from the network, and design their codebooks accordingly.

More specifically, the modified network coding scheme operates as follows. Users are required to split their bit streams over the middle tree into two sub-streams: a network-codable substream X_i^{NC}, and a routable substream, X_i^R. The network-codable substream must satisfy the conditional entropy requirement

$$H(X_i^{NC}|X_i^{SUO}, X_i^{SB}) = 0 \tag{7}$$

This guarantees that the network coded bits will be decodable at both receivers. Users then submit bids to the network on their total block length N_i, and the block length of their network codable symbol, W_i. The network then determines block lengths for the network codable and routable symbols of each user as follows:

$$N = \text{LCM}(N_1, N_2) = k_1 N_1 = k_2 N_2$$

$$W = \min(k_1 N_1 w_1, k_2 N_2 w_2)$$

$$r_1 = \lfloor \alpha(N - w) \rfloor$$

$$r_2 = N - W - \lfloor \alpha(N - w) \rfloor$$

where N is the super-block length (similar to the original formulation in Section II), W is the block length of the network-codable stream for each user, and r_1 and r_2 are the block lengths of the routable streams for users 1 and 2, respectively. The value α is a constant chosen apriori by the network. Users are then given these four parameters, and allowed to design their codebooks autonomously. In the following subsections, we will see that this game results in a capacity-achieving, dominant strategy equilibrium.

A. Game Formulation

Formally, we denote the non-cooperative network coding game by $\mathcal{F} = [\mathcal{N}, \{\sigma_i\}, \{U_i(\cdot)\}]$. As in Section III, $\mathcal{N} = \{1, 2\}$ is the set of players, where $S(i)$ and $T(i)$ are the source and destination nodes for unicast flows $i = 1, 2$. The strategy space of player i is denoted by Σ_i, and consists of all possible strategies $\sigma_i = \{N_i, W_i, B_i, f_i(\cdot)\}$ where $N_i \geq 0$, $W_i \geq 0$, $B_i \geq 0$, and $f_i(\cdot)$ satisfy the link capacity constraints given by (1)-(3) and the conditional entropy constraint given by (7). Finally, we model the utility of user i as $U_i(\sigma_i, \sigma_{-i}) = \sum_{j=1}^{B_i} 1(b_{i,j})$, where

$$1(b_{i,j}) = \begin{cases} 1 & \text{if } b_{i,j} \text{ is correctly decoded} \\ 0 & \text{else} \end{cases}$$

is an indicator function for information bit $b_{i,j}$ being correctly decoded at its own receiver.

B. Dominant Strategy Equilibrium

It turns out that it is possible to achieve capacity of the network in a dominant strategy sense. Consider the strategies (σ_1^*, σ_2^*), where σ_i is as follows:

$$\begin{aligned} N_i^* &= N_i \\ W_i^* &= (1 - c_i)N_i \\ B_i^* &= W + r_i + NC_i \\ X_i^{*NC} &= [b_{i,1}, \ldots, b_{i,W}] \\ X_i^{*R} &= [b_{i,W+1}, \ldots, b_{i,W+r_i}] \\ X_i^{*SUS} &= [b_{i,W+r_i+1}, \ldots, b_{i,W+r_i+Nc_i}] \\ X_i^{*SUO} &= [b_{i,1}, \ldots, b_{i,W}] \end{aligned}$$

Then, we have the following:

Theorem 2: Consider strategies (σ_1^*, σ_2^*) as given above. This strategy profile is a dominant strategy, and achieves capacity of the network.

Proof:

To see that these strategies achieve capacity, we note that all of the information bits sent by users 1 and 2 are decodable at the receivers. Thus, we have

$$\begin{aligned} B &= B_1^* + B_2^* \\ &= W + r_1 + NC_1 + W + r_2 + NC_2 \\ &= 2W + r_1 + r_2 + k_1 c_1 + k_2 c_2 \\ &= 2\min(k_1(1 - c_1)N_1, k_2(1 - c_2)N_2) \\ &\quad + \lfloor \alpha(N - W) \rfloor + N - W - \lfloor \alpha(N - W) \rfloor + Nc_1 + Nc_2 \\ &= N(2\min(1 - c_1, 1 - c_2) + c_1 + c_2 \\ &= N(2 + \min(c_1, c_2)) \end{aligned}$$

which is the sum-rate capacity of the network.

In the interest of space, we do not include the proof of dominant strategies here. It follows along the lines of the dominant strategy proof give in [7] for flow-based models of network coding.

■

V. Conclusions and Future Work

In this paper, we formulated a non-cooperative network coding game in which users strategies are their codebook designs. We have shown that when the network performs a blind XOR operation, it is difficult to construct strategies that are both capacity-achieving and can be selected in a distributed manner at the source nodes. By changing the network coding scheme, however, it is possible to construct a dominant-strategy equilibrium that is also capacity-achieving.

Although the results presented here are exciting, they are but a first step in the study of network coding games. In this paper, we have focused on a particular network. Although these results can be generalized to a broader class two-user unicast networks with similar properties, we do not have results for more general unicast networks.

In addition, although we have shown the existence of multiple Nash equilibrium in the blind network coding case, we have not shown that there does not exist a dominant strategy equilibrium. The challenge in constructing a dominant strategy equilibrium is the fact that the interference a user sees is dependant both on the network coding scheme, and on the codebook design of the other user. If the other user's codebook is not known apriori, the user is coding against an unknown channel. One way to address this is to formulate alternate utility functions, in which users are interested in vanishing probability of error for a large subset of possible codebooks. Another approach would be to allow each user to have a large but finite number of possible codebooks, each of which has some probability associated with it. We can then examine the outcome of Bayesian games to see what types of equilibrium emerge.

Finally, there is the question of whether capacity-achieving Nash equilibrium can be found for all deterministic interference channels. Berry and Tse have shown in [8] that there exist capacity-achieving Nash equilibrium for a deterministic model of a Gaussian interference channel. Similarly, we have shown that there exist capacity-achieving Nash equilibrium for the deterministic channel induced by a network coding game. It will be interesting to investigate whether these are simply two special cases, or whether there is an underlying result that ties these cases together.

Appendix A
Proof of Theorem 1

In order to show that all of the points in the region $\mathcal{C}\cap\Delta$ can be achieved as Nash equilibria, we construct coding schemes that achieve each of the corner points as a Nash equilibrium. We then use a time-division argument to show that all convex combinations (and hence the entire region) is achievable as a Nash equilibrium.

Lemma 1: The points $(R_1, R_2) = (1+c_1, 1+\min(c_1, c_2)-c_1)$ and $(R_1, R_2) = (1+\min(c_1, c_2)-c_2, 1+c_2)$ are achievable as Nash equilibria.

Proof: First, consider the point $(R_1, R_2) = (1 + c_1, 1 + \min(c_1, c_2) - c_1)$. Let σ_1 be as follows:

$$\begin{aligned}
N_1 &= N \\
B_1 &= N(1+c_1) \\
X_1^{MUS} &= [b_{1,1}, \ldots, b_{1,N}] \\
X_1^{SUS} &= [b_{1,N+1}, \ldots, b_{1,N(1+c_1)}] \\
X_1^{SUO} &= [b_{1,1}, \ldots, b_{N(1-\max(c_1,c_2))}]
\end{aligned}$$

Let σ_2 be as follows:

$$\begin{aligned}
N_2 &= N \\
B_2 &= N(1-\max(c_1,c_2)) + Nc_2 \\
X_2^{MUS} &= [b_{2,1}, \ldots, b_{2,N(1-\max(c_1,c_2))}] \\
X_2^{SUS} &= [b_{2,N(1-max(c_1,c_2))+1}, \ldots, b_{2,N(1-max(c_1,c_2)+Nc_2)}] \\
X_2^{SUO} &= [b_{2,1}, \ldots, b_{2,N(1-\max(c_1,c_2))}]
\end{aligned}$$

First, fix σ_2 and consider user 1. Given the strategy profile (σ_1, σ_2), user 1 can correctly decode all of its $N(1 + c_1)$ information bits. However, from the max-flow min-cut theorem (see e.g. [9]), we know that this is the most decodable information bits that user 1 can have. Hence, given σ_2, there is no strategy that outperforms σ_1.

Now, fix σ_1 and consider user 2. Given the strategy profile (σ_1, σ_2), user 2 can correctly decode all of its $N(1 - \max(c_1, c_2)+c_2)$ information bits. However, we know that the capacity of user 2 is upper bounded by the mutual information of the symbols it sends and the symbols it receives [10]. This gives

$$\begin{aligned}
C_2 &\leq I(X_2^{MUS}, X_2^{SUS}, X_2^{SUO}; X_2^{SUS}, X_1SUO, Z) \\
&= H(X_2^{SUS}, X_1^{SUO}, Z) \\
&\quad -H(X_1^{SUO}, Z|X_2^{MUS}, X_2^{SUS}, X_2^{SUO}) \\
&= H(X_2^{SUS}, X_1^{SUO}, Z) - H(X_1^{SUO}, X_1^{MUS}) \\
&\leq H(X_2^D) + H(Z|X_1^{SUO}) - H(X_1^{MUS}|X_1^{SUO}) \\
&\leq Nc_2 + N - N\max(c_1, c_2)
\end{aligned}$$

Hence, given σ_1, there is no strategy that outperforms σ_2, and the point is a Nash equilibrium.

By symmetry, $(R_1, R_2) = (1 + \min(c_1, c_2) - c_2, 1 + c_2)$ is also a Nash equilibrium.

■

Lemma 2: The points $(R_1, R_2) = (1 + c_1, c_2)$ and $(R_1, R_2) = (c_1, 1 + c_2)$ are achievable as Nash equilibria.

Proof: First, consider the point $(R_1, R_2) = (1 + c_1, c_2)$. Let σ_1 be as follows:

$$\begin{aligned} N_1 &= N \\ B_1 &= N(1+c_1) \\ X_1^{MUS} &= [b_{1,1}, \ldots, b_{1,N}] \\ X_1^{SUS} &= [b_{1,N+1}, \ldots, b_{1,N(1+c_1)}] \\ X_1^{SUO} &= \emptyset \end{aligned}$$

Let σ_2 be as follows:

$$\begin{aligned} N_2 &= N \\ B_2 &= Nc_2 \\ X_2^{MUS} &= \emptyset \\ X_2^{SUS} &= [b_{2,1}, \ldots, b_{2,Nc_2}] \\ X_2^{SUO} &= \emptyset \end{aligned}$$

First, fix σ_2 and consider user 1. Given the strategy profile (σ_1, σ_2), user 1 can correctly decode all of its $N(1 + c_1)$ information bits. However, from the max-flow min-cut theorem (see e.g. [9]), we know that this is the most decodable information bits that user 1 can have. Hence, given σ_2, there is no strategy that outperforms σ_1.

Now, fix σ_1 and consider user 2. Given the strategy profile (σ_1, σ_2), user 2 can correctly decode all of its Nc_2 information bits. However, we know that the capacity of user 2 is upper bounded by the mutual information of the symbols it sends and the symbols it receives [10]. This gives

$$\begin{aligned} C_2 &\leq I(X_2^{MUS}, X_2^{SUS}, X_2^{SUO}; X_2^{SUS}, X_1 SUO, Z) \\ &= H(X_2^{SUS}, X_1^{SUO}, Z) \\ &\quad - H(X_1^{SUO}, Z | X_2^{MUS}, X_2^{SUS}, X_2^{SUO}) \\ &= H(X_2^{SUS}, X_1^{SUO}, Z) - H(X_1^{SUO}, X_1^{MUS}) \\ &\leq H(X_2^{D}) + H(Z|X_1^{SUO}) - H(X_1^{MUS}|X_1^{SUO}) \\ &\leq Nc_2 + N - H(X_1^{MUS}) \\ &= Nc_2 \end{aligned}$$

Hence, given σ_1, there is no strategy that outperforms σ_2, and the point is a Nash equilibrium.

By symmetry, $(R_1, R_2) = (c_1, 1 + c_2)$ is also a Nash equilibrium. ■

Lemma 3: The point $(R_1, R_2) = (c_1, c_2)$ is achievable as a Nash equilibrium.

Proof: Let σ_1 be as follows:

$$\begin{aligned} N_1 &= N \\ B_1 &= N(1+c_1) \\ X_1^{MUS} &= [b_{1,1}, \ldots, b_{1,N}] \\ X_1^{SUS} &= [b_{1,N+1}, \ldots, b_{1,N(1+c_1)}] \\ X_1^{SUO} &= \emptyset \end{aligned}$$

Let σ_2 be as follows:

$$\begin{aligned} N_2 &= N \\ B_2 &= N(1+c_2) \\ X_2^{MUS} &= [b_{2,1}, \ldots, b_{2,N}] \\ X_2^{SUS} &= [b_{2,N+1}, \ldots, b_{2,N(1+c_2)}] \\ X_2^{SUO} &= \emptyset \end{aligned}$$

First, fix σ_2 and consider user 1. Given the strategy profile (σ_1, σ_2), user 1 can correctly decode only Nc_1 information bits. However, from the proof of Lemma 2 we can see that this is the upper bound on capacity of user 1 for σ_2.

Similarly, if we fix σ_1 and consider user 2, we see that user 2 can correctly decode only Nc_2 information bits. Again from the proof of Lemma 2 we can see that this is the upper bound on capacity of user 2 for σ_1.

Hence, the strategy profile (σ_1, σ_2) is a Nash equilibrium. ■

Having constructed the five corner points of the region $\Delta_{\mathcal{NE}}$, we can use a time-division like argument to show that any convex combination of these five corner points (hence the entire region) is achievable as a Nash equilibrium, similar to that described in [8].

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