Opportunistic Routing with Congestion Diversity in Wireless Multi-hop Networks

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***Abstract*—This paper considers the problem of routing packets across a multi-hop network consisting of multiple sources of traffic and wireless links with stochastic reliability while ensuring bounded expected delay. Each packet transmission can be overheard by a random subset of receiver nodes among which the next relay/router is selected opportunistically. The main challenge in the design of minimum-delay routing policies is balancing the trade-off between routing the packets along the shortest paths to the destination and controlling the congestion and distributing traffic uniformly across the network. Simple opportunistic variants of shortest path routing may, under heavy traffic scenarios, result in severe congestion and unbounded delay. While the opportunistic variants of backpressure, which ensure a bounded expected delay, are known to exhibit extremely poor delay performance at low to medium traffic conditions.**

Combining important aspects of shortest path routing with those of backpressure routing, this paper provides an opportunistic routing policy with congestion diversity (ORCD). ORCD uses a measure of draining time to opportunistically identify and route packets along the paths with an expected low overall congestion. Using a novel Lyapunov function construction, ORCD is proved to ensure a bounded expected delay for all networks and under any admissible traffic (without any knowledge of traffic statistics). Furthermore, the expected delay encountered by the packets in the network under ORCD is compared against known existing routing policies via simulations and substantial improvements are observed. Finally, the paper proposes practical implementations and discusses criticality of various assumptions in the analysis.

I. Introduction

Opportunistic routing for multi-hop wireless ad-hoc networks has seen recent research interest to overcome deficiencies of conventional routing [1]–[5]. Opportunistic routing mitigates the impact of poor wireless links by exploiting the broadcast nature of wireless transmissions and the path diversity. More precisely, the routing decisions are made in an online manner by choosing the next relay based on the actual transmission outcomes as well as a rank ordering of neighboring nodes. The authors in [4] provided a Markov decision theoretic formulation for opportunistic routing. In particular, it is shown that for any given packet and at any relaying epoch, the optimal routing decision, in the sense of minimum cost or hop-count, is to select the next relay node based on an index. This index is equal to the expected cost or hop-count of relaying the packet along the least costly or the shortest feasible path to the destination. As such, [4] provides a unifying framework for almost all versions of opportunistic routing [1]–[3], where the variations are due to the authors' choices of costs; e.g. for ExOR [3], the cost to be minimized is the expected hop-counts (ETX).

When multiple streams of packets are to traverse the network, however, it might be necessary to route some packets along longer paths, if these paths eventually lead to links that are less congested. More precisely, and as noted in [6], [7], the opportunistic routing schemes in [1]–[5] can potentially cause severe congestion and unbounded delays (see examples given in [7]). In contrast, it is known that a simple routing policy, known as backpressure [8], ensures bounded expected total backlog for all stabilizable arrival rates. In the opportunistic context, diversity backpressure routing (DIVBAR) [6] provides an opportunistic generalization of backpressure which incorporates the wireless diversity. To ensure throughput optimality, backpressure-based algorithms [6], [8] do something very different from [1]–[5]; rather than any metric of closeness to the destination (or cost), they choose the receiver with the largest positive differential backlog (routing responsibility is retained by the transmitter if no such receiver exists). This very property of ignoring the cost to the destination, however, becomes the bane of this approach, leading to poor delay performance (see [6], [7]).

Combining the congestion information with the shortest path calculations proposed in [4], this paper provides an opportunistic routing policy with congestion diversity (ORCD). ORCD is shown to be throughput optimal and hence, it guarantees bounded expected delay. ORCD exhibits better delay performance than existing backpressure-based routing policies, since it routes packets along the paths with minimum overall congestion.

The remainder of this paper is organized as follows. In Section II we discuss the network model and formulate the problem. Section III introduces our proposed routing algorithm, ORCD, and states the main analytical and simulation results of the paper. In Section IV, we discuss various implementation issues and extensions of ORCD. The non-idealities and assumptions in the network model are addressed and explained in Section V. Finally, we conclude the paper and discuss future work in Section VI.

We close this section with a note on the notations used. Let $[x]^+ = \max\{x, 0\}$. The indicator function $\mathbf{1}_{\{X\}}$ takes the value 1 whenever event X occurs, and 0 otherwise. For any set S,

$|S|$ denotes the cardinality of S, while for any vector $\boldsymbol{v}$, $\|\boldsymbol{v}\|$ denotes the euclidean norm of $\boldsymbol{v}$. For any set S, $int(S)$ is the set of all interior points of S. When dealing with a sequence of sets $C_1, C_2, \ldots$, we define $C^i = \cup_{j=1}^{i} C_j$.

II. PROBLEM SETUP

A. Network Model

We consider a time slotted system with slots indexed by $t \in \{0, 1, 2, \ldots\}$ where slot t refers to the time interval $[t, t+1)$. There are $N+1$ nodes in the network labeled by $\Omega = \{0, 1, \ldots, N\}$, where node 0 is assumed to be the destination.

Let $A_i(t)$ represent the amount of data that exogenously arrives to node i during time slot t. Arrivals are assumed to be i.i.d. over time and bounded by a constant A_{max}. Let $\lambda_i = \mathbb{E}[A_i(t)]$ denote the exogenous arrival rate to node i. We define $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_N]$ to be the arrival rate vector.

A "routing decision cycle" consists of three stages in sequence: (a) transmission, (b) acknowledgment, and (c) relaying. During the transmission stage, node i transmits a maximum of one packet. During the acknowledgment stage each node that has successfully received the transmitted packet, sends an acknowledgment (ACK) to node i. During the relaying stage, the relaying responsibility of the packet is shifted to one of the nodes that have received this packet or is retained by node i. This decision is announced by node i using a forwarding (FO) message. We assume that the acknowledgment and relaying stages are error free, while packet transmissions in general encounter link failures. Let p_{ij} be the probability that the packet transmitted by node i is successfully received by node j. Moreover, we assume that each network node transmits over an orthogonal channel, so that there is no inter-channel interference and the links are independent. Let $S_i(t)$ represent the (random) set of nodes that have received the packet transmitted by node i at time slot t. We refer to $S_i(t)$ as the set of *potential forwarders* for node i. Note that due to links independent realization, for all $S \subseteq \Omega$, $P(S_i(t) = S \mid i \text{ transmits at time } t) = \prod_{k\in S} \prod_{l \notin S} p_{ik}(1-p_{il})$.

We define a *routing decision* $\mu_{ij}(t)$ to be the number of packets whose relaying responsibility is shifted from node i to node j during time slot t ($\mu_{ii}(t) = 1$ means that i retains the packet). Note that $\mu_{ij}(t)$ forms the departure process from node i, while it creates an endogenous arrival to node j, and hence,

$$\mu_{ij}(t) \in \{0,1\}\ ,\ \mu_{ij}(t) \le \mathbf{1}_{\{j \in S_i(t)\}}\ ,\ \sum_{j=0}^{N} \mu_{ij}(t) = 1. \quad (1)$$

Without loss of optimality, we assume that $p_{ii} = 1$ and $\mu_{i0}(t) = 1$ if $0 \in S_i(t)$.

Definition. A *routing policy* is a collection of causal routing decisions $\cup_{i,j\in\Omega} \cup_{t=0}^{\infty} \{\mu_{ij}(t)\}$.[1]

[1]For all $i, j \in \Omega$ and $\theta \in \{0,1\}$, the decision $\{\mu_{ij}(t) = \theta\}$ belongs to the σ-field generated by $\cup_{i,j\in\Omega}\{A_i(0), S_i(0), \mu_{ij}(0), \ldots, A_i(t-1), S_i(t-1), \mu_{ij}(t-1), A_i(t), S_i(t)\}$.

B. Queue Backlogs, Delay, and Throughput Optimality

We assume packets that arrive exogenously at node i as well as packets routed to node i from other nodes are queued at node i in a buffer with infinite storage for future transmissions. Let $Q_i(t)$ denote the queue backlog of node i at time slot t. We assume any data that is successfully delivered to the destination will exit the network and hence, $Q_0(t) = 0$ for all time slots t. We define $\boldsymbol{Q}(t) = [Q_1(t), Q_2(t), \ldots, Q_N(t)]$ to be the vector of queue backlogs of nodes $1, 2, \ldots, N$.

The selection of routing decisions under policy Π together with the exogenous arrivals impact the queue backlog of node i, $i \in \Omega$, in the following manner:

$$\begin{aligned} Q_i^{\Pi}(t+1) =& [Q_i^{\Pi}(t) - \sum_{j\in\Omega} \mu_{ij}(t)]^+ \\ &+ \sum_{j\in\Omega} \mu_{ji}(t) \mathbf{1}_{\{Q_j^{\Pi}(t) \ge \mu_{ji}(t)\}} + A_i(t), \end{aligned} \quad (2)$$

where the superscript Π emphasizes the dependence of queue backlog process on the choice of policy Π.

From Little's Theorem, we know that in a stationary and ergodic setting, the average delay experienced by the packets under policy Π is proportional to the total queue backlog, i.e.

$$\mathbb{E}[\text{delay}] \propto \mathbb{E}[Q_{tot}^{\Pi}(t)] = \mathbb{E}[\sum_{i=1}^{N} Q_i^{\Pi}].$$

It is natural, then, to search for a policy Π with small $\mathbb{E}[\sum_{i=1}^{N} Q_i^{\Pi}]$. Unfortunately, finding a delay optimal policy Π^* under general multi-hop network topologies remains an open problem even in the presence of perfect information about the network and arrival process. A more modest research goal, thus, is to find a policy which can ensure a bounded total backlog, $\mathbb{E}[\sum_{i=1}^{N} Q_i^{\Pi}]$.[2]

Definition. Given an ergodic exogenous arrival process with rate $\boldsymbol{\lambda}$, a routing policy Π is said to *stabilize* the network if $Q_{tot}^{\Pi}(t)$ is ergodic and $\mathbb{E}[Q_{tot}^{\Pi}(t)]$ remains bounded when packets are routed according to Π. The *stability region* of the network (denoted by $\mathfrak{S}$) is the set of all arrival rate vectors $\boldsymbol{\lambda}$ for which there exists a routing policy that stabilizes the network.

Definition. A routing policy is said to be *throughput optimal* if it stabilizes the network for all arrival rate vectors that belong to the interior of the stability region.

C. Prior Work

The goal of this paper is to design a routing policy that improves the delay performance over existing routing policies while ensuring throughput optimality. The main challenge in the design of minimum-delay routing policies is balancing the trade-off between two goals of routing packets along the shortest paths to the destination and controlling the traffic congestion and distributing traffic uniformly across the network.

[2]In Markovian setting, this policy is also referred to one that ensures the positive recurrence of the queue backlog Markov chain, $\boldsymbol{Q}(t)$.

Opportunistic variants of shortest path routing [1]–[4] may, under heavy traffic scenarios, result in unbounded $\mathbb{E}[Q_{tot}(t)]$. While to the best of our knowledge, all existing (provably) throughput optimal routing policies [6], [8]–[13] (those for which $\mathbb{E}[Q_{tot}(t)] < \infty$) distribute the traffic locally in every neighborhood and hence, result in large $\mathbb{E}[Q_{tot}(t)]$. We attribute this shortcoming to the popular method of proof in the literature: In [6], [8]–[13], various throughput optimal routing policies are identified by constructing various Lyapunov functions and their maximum negative drift. Due to the topology-independent and simple structure of the proposed Lyapunov functions in the literature, such as quadratic in case of [6], [8]–[11], exponential in case of [12], and logarithmic in case of [13], [14], the obtained routing solutions inherit and exhibit a localized and topology-independent nature.

In [6], elements of shortest path computations are used to arrive at an enhanced version of DIVBAR (E-DIVBAR) in an attempt to mitigate the shortcomings of the two approaches. When choosing the next relay among the set of potential forwarders, E-DIVBAR considers the sum of the differential backlog and the expected hop-count to the destination (also known as ETX). However, and as shown in [7], E-DIVBAR does not guarantee a better delay performance than DIVBAR.

In [15], perhaps the most related work to ours, the authors consider a flow-level model of the network and propose a routing policy referred to as *min-backlogged-path* routing. Under min-backlogged-path routing, the flow which arrives at node i and destined for node j, is routed along the path with minimum total backlog from i to j. However, the throughput optimality of this policy is only shown under unrealistic assumptions on the utilization of every link. Furthermore, the routing decisions require an enumeration of paths across the network.

III. Main Results

In this section, we first describe the workings of ORCD. Then we state the major result of this paper which is the throughput optimality of ORCD. Lastly, the expected delay performance of ORCD is compared against other existing routing policies via simulations.

A. Opportunistic Routing with Congestion Diversity (ORCD)

In ORCD nodes are ordered according to a cost measure of congestion "down the stream" from each node i denoted by $V_i(t)$ which satisfy the following fixed point equation for all time slots t:

$$V_0(t) = 0, \tag{3}$$
$$V_i(t) = Q_i(t) + \sum_{S\subseteq\Omega} \big(\prod_{k\in S} \prod_{l\notin S} p_{ik}(1-p_{il}) \big) \min_{j\in S} V_j(t). \tag{4}$$

ORCD makes routing decisions $\{\mu^*_{ij}(t)\}_{i,j\in\Omega}$ such that $\mu^*_{ij}(t) = 1$ only when $j \in S_i(t)$ and $V_j(t) \le V_k(t)$ for all $k \in S_i(t)$.

Note that if $\boldsymbol{Q}(t)$ is stationary, $V_i(t)$ can be interpreted as the expected delay for a packet arriving at node i till it reaches the destination. Thus, ORCD, can be interpreted as the policy in which packets are opportunistically sent along the least congested paths.

The computation of $V_i(t)$, hence, the complexity and implementation issues of ORCD are discussed in Section IV.

B. Analytical Result

In this section, we provide the main analytical result of this paper.

Theorem 1. *ORCD is throughput optimal.*

By Theorem 1, under ORCD, the average total queue backlog remains bounded. Little's theorem implies that under ORCD, expected delay is bounded.

The proof of Theorem 1 is based on the Foster-Lyapunov Theorem and is available in [16]. However, the structure of the Lyapunov function and sketch of the proof is provided in the appendix.

C. Simulation Result

In this section, we compare the expected delay encountered by the packets in the network under various routing policies: ORCD, DIVBAR, and E-DIVBAR. Additionally, the simulations include the performance of D-ORCD, a heuristic but distributed modification of ORCD whose detailed workings and structure are given in Subsection IV-D. Simulations are done for the network of Fig. 1.

The network shown in Fig. 1 is parameterized by three parameters, N, K, and M. Similar to the structure studied in [17], nodes $A_1, A_2, \ldots, A_N$ form a "hole" in the network whose size is controlled by parameter N. On the other hand, parameters M and K control the path diversity on the right-hand side of the network. In other words, the choice of the network shown in Fig. 1 enables the simulations to clearly reveal the high capability of ORCD in balancing the traffic, avoiding "dead-ends" and "holes," and taking advantage of path diversity in the network.

Figures 2- 4 illustrate the delay gains under ORCD and D-ORCD as parameters, N, M, and λ_S are varied. The following variables are fixed for all the simulations: $\lambda_C = 0.8, \lambda_A = \lambda_{A_i} = \lambda_B = \lambda_{B_i} = 0, \lambda_{B_{ij}} = 0.5$. Note that the source can route packets either through node A or node B. Since only node S has a routing choice and for illustration purposes, we focus on the delay experienced by packets originating in node S when plotting Figures 2- 4.

Fig. 2 provides the expected delay encountered by the source packets under various routing policies, as the size of the "hole," N, increases. Since node C has a high arrival rate, under DIVBAR, the packets that arrive at node A from source are likely to be forwarded to A_i, $i \in \{1, 2, \ldots, N\}$. Due to the structure of this "hole," these packets wonder between nodes $A_1, A_2, \ldots, A_N$ a non-negligible time before their return to A and eventual forwarding to C. This results in a loss of performance for DIVBAR which increases with the size of the "hole." In other words, the size of the "hole" in the network significantly impacts the performance of backpressure-based policies. In contrast, increasing N has no effect on

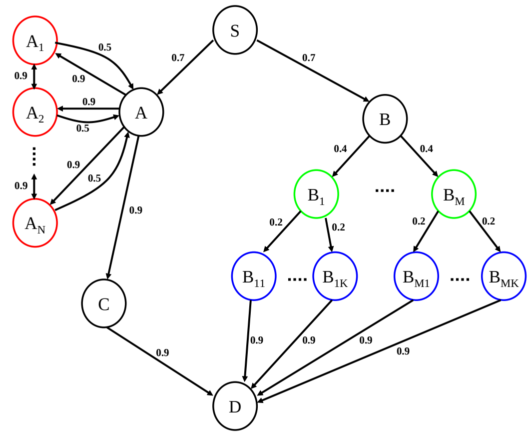

Fig. 1. Structure of the network. The fractions on the links show the probability of successful transmission on each link.

the performance of ORCD and D-ORCD. This is because $V_A(t) < V_{A_i}(t)$, $i = 1, 2, \ldots, N$, for all time slots t, in effect, preventing the packets to enter the "hole."

Note that E-DIVBAR, in this scenario, has a performance that is strictly dominated by DIVBAR, as the choice of the total sum of ETX and the backlog creates a further bias in routing the packets through node A, hence, exhibiting a worse delay performance. Note that the value of ETX utilized by E-DIVBAR at nodes A and B are as follows:

$$\begin{aligned} ETX(A) &= \frac{1}{0.9} + \frac{1}{0.9} = 2.22, \\ ETX(B) &= \frac{1}{0.4} + \frac{1}{0.2} + \frac{1}{0.9} = 8.61. \end{aligned}$$

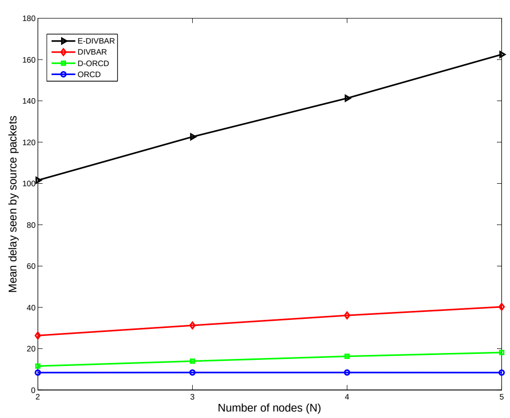

Fig. 2. Mean Delay seen by source packets in the network of Fig. 1 as the size of the "hole," N, varies. The plots are given for $K = 2$, $M = 2$, and $\lambda_S = 0.2$

Fig. 3 shows the effect of changing parameter M on the delay performance of the routing policies. As the path diversity increases in the network, i.e. with an increase in M, the average delay under all policies decreases, with the improvements more significant under DIVBAR.

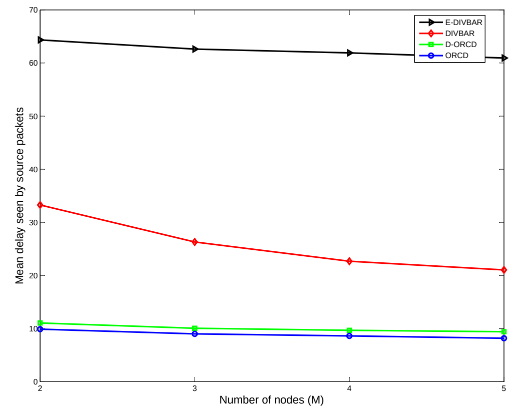

Fig. 3. Mean Delay seen by source packets in the network of Fig. 1 as the number of available paths via node B, M, varies. The plots are given for $\lambda_S = 0.4$, $K = 2$, $N = 2$.

The performance improvements under ORCD is not limited to the low traffic scenarios. Fig. 4 illustrates the delay performance of ORCD along with other routing policies under different traffic conditions. Note the improved performance of E-DIVBAR with increased traffic intensity from node S, λ_S. In high traffic scenarios, the path to the destination via node A becomes more congested, making E-DIVBAR less sensitive to the ETX metric, while increasing the number of packets wondering around in the A_1-A_N "hole." These two factors result in a closing of the performance gap between E-DIVBAR and DIVBAR. It is critical to note that because of the inherent path diversity available at node B, the delay under ORCD and D-ORCD remains robust against an arrival rate increase.

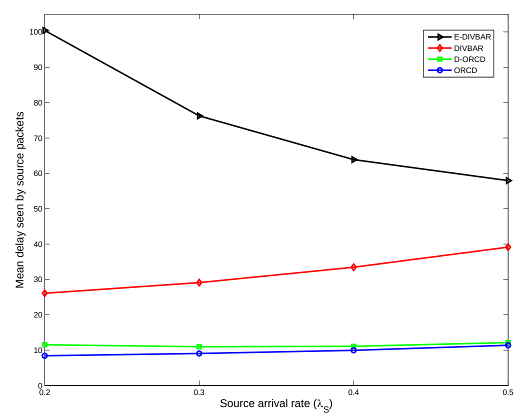

Fig. 4. Mean Delay seen by source packets in Network of Fig. 1 under different traffic conditions. The plots are given for $K = 2$, $M = 2$, and $N = 2$

In the next section, we discuss the issues associated with computation of the time-varying congestion cost measures $V_i(\cdot)$, $i \in \Omega$. Furthermore, we propose practical implementations and heuristics.

IV. Practical Implementation of ORCD

A. Three-way Handshake

As mentioned in Subsection II-A, routing decisions are made via a three-way handshake (transmission, acknowledgment, and relaying). This three-way handshake can be easily implemented via a set of ACK and FO packets with low overhead. Alternatively, the handshake can be implemented via a combination of predetermined timing for the reception of ACKs and appending FO to the data packets. In other words, each transmitting node waits for a certain amount of time to collect acknowledgments from the recipients. Upon selection of the next forwarder opportunistically, it then appends the forwarder's id to the header of the next data packet to be transmitted. The impact of unreliable channel or delay is discussed in Subsection V-D.

B. Computation of $V_i(t)$'s

1) Centralized Computation: Congestion cost measures $V_i(t)$, $i \in \Omega$, can be calculated for each time slot t using a Dijkstra-type algorithm as follows:

1. Initialize
   - $V_0(t) = 0$ where 0 is the index of the destination.
   - $V_i(t) = \infty$ for all nodes i but the destination.
   - $\mathcal{A} = \{0\}$. Let $\mathcal{A}^c$ be the complement of $\mathcal{A}$ with respect to Ω.
2. For all nodes $i \in \mathcal{A}^c$, calculate $J_i(t)$ as follows:

$$J_i(t) = \frac{Q_i(t)}{P(i,t)} + \sum_{k\in\mathcal{A}} \frac{P(i,k,t)}{P(i,t)} V_k(t), \quad (5)$$

   where

$$P(i,t) = \sum_{S: S\cap\mathcal{A}\neq\emptyset} \Big(\prod_{l\in S} p_{il}\Big)\Big(\prod_{l\notin S}(1-p_{il})\Big), \quad (6)$$

$$P(i,k,t) = \sum_{\substack{S: k\in S\cap\mathcal{A},\\ V_k(t)<V_j(t)\ \forall j\in S}} \Big(\prod_{l\in S} p_{il}\Big)\Big(\prod_{l\notin S}(1-p_{il})\Big). \quad (7)$$

3. Update
   - Find the node with the minimum value of $J_i(t)$:

$$i^* = \arg\min_{i\in\mathcal{A}^c} J_i(t).$$

   - $V_{i^*}(t) = J_{i^*}(t)$.
   - $\mathcal{A} = \mathcal{A} \cup i^*$.
4. Repeat Steps 2 and 3 till $\mathcal{A} = \Omega$.

Remark This algorithm has a worst-case run time of $O(N^2)$.

2) Iterative and Decentralized Computation: Congestion cost measures $V_i(t)$, $i \in \Omega$, can also be calculated using an iterative algorithm at each node i in a decentralized manner as follows:

$$V_i^k(t) = Q_i(t) + \sum_{S\subseteq\Omega} \Big(\prod_{l\in S} p_{il}\Big)\Big(\prod_{l\notin S}(1-p_{il})\Big) \min_{j\in S} V_j^{k-1}(t). \quad (8)$$

Remark It is shown in [4], [18] that $\lim_{k\to\infty} V_i^k(t) = V_i(t)$. This algorithm has a worst-case complexity of $O(N)$ at any node and allows for decentralized operation, in which any node i's computation only relies on communication with immediate neighbors and knowledge of p_{ij}, $j \in \Omega$.

C. Infrequent Computations and Outdated Queue State

In this part, we will show that ORCD can be implemented without loss of throughput optimality by using outdated backlog information with the frequency of computation small in comparison to the length of any routing decision cycle. More precisely, let Infreq-ORCD be a modified version of ORCD in which the computation of (an approximate) index $\hat{V}_i$ is done every T slots to solve the fixed point equation:

$$V_i(nT) = Q_i((n-1)T) + \sum_{S\subseteq\Omega} \Big(\prod_{k\in S}\prod_{l\notin S} p_{ik}(1-p_{il})\Big) \min_{j\in S} V_j(nT), \quad (9)$$

and the relaying decisions at time $nT \leq t < (n+1)T$, are based on $\hat{V}_i(t) = V_i(nT)$, $i \in \Omega$.

Remark The nature of Infreq-ORCD is such that it allows for practically low computational complexity and/or decentralized implementation. For instance, when T is large, the decisions of the centralized controller in Subsection IV-B1 can be effectively flooded back to the source nodes from the destination. Similarly, for sufficiently large T, the iterative algorithm in Subsection IV-B2 can be used, where nodes use outdated $\boldsymbol{Q}(nT)$ to compute $\hat{V}_i(t)$ for all $nT \leq t < (n+1)T$.

Theorem 2. *Infreq-ORCD is throughput optimal.*

Proof: Proof is provided in the appendix. ■

D. Asynchronous and Distributed ORCD

In this section, we combine the decentralized nature of (8) with a salient feature of Infreq-ORCD, i.e. its use of outdated information, to arrive at an asynchronous and distributed routing algorithm (AD-ORCD) in which routing decisions are made according to the ordering dictated by an approximate value function $\hat{V}_i(t)$, $i \in \Omega$, where

$$\hat{V}_i(t) = Q_i(t) + \sum_{S\subseteq\Omega} \Big(\prod_{k\in S}\prod_{l\notin S} p_{ik}(1-p_{il})\Big) \min_{j\in S} \hat{V}_j(t-T_j), \quad (10)$$

and where $t - T_j$ is the most recent time that node i received an update from node j.

When $T_j = 1$ for all $j \in \Omega$, the algorithm is distributed but synchronous, which we call D-ORCD and whose performance was investigated in Subsection III-C.

V. Discussion of Model

A. Probabilistic Link Model

In this paper we characterized the behavior of the wireless network using a simple probabilistic model. In particular, we used p_{ij} to denote the probability of successful transmission from node i to node j and links were assumed to be independent. However, this independent link model is only chosen for ease of exposition and our work can easily be extended to a probabilistic model capturing inter-link dependence. More precisely, the behavior of the wireless channel can be captured by a probabilistic *local broadcast model* [4] of transition probabilities $P(S|i)$, $S \subseteq \Omega$, $i \in \Omega$, where $P(S|i)$ denotes the probability of successful reception of the packet transmitted from node i by all the nodes in S. Note that for all $S \neq S'$, successful reception at S and S' are mutually exclusive and $\sum_{S \subseteq \Omega} P(S|i) = 1$.

In this new setting, the fixed point equation (4) can be generalized to

$$V_i(t) = Q_i(t) + \sum_{S \subseteq \Omega} P(S|i) \min_{j \in S} V_j(t). \tag{11}$$

Similarly, in the algorithms provided in Subsection IV-B for the computation of $V_i(t)$'s, the term $\left(\prod_{l \in S} p_{il}\right)\left(\prod_{l \notin S}(1 - p_{il})\right)$ is replaced by $P(S|i)$.

Given the dependence of our formulation and structure of proposed policies and algorithms on the probabilistic local broadcast model, one might wonder about the validity of the work in the face of modelling errors, or lack of off-line models. It is straight forward to show that the throughput optimality of ORCD is robust to all channel estimation errors. However, erroneous link models, in general, can significantly degrade the delay performance of the algorithm.

B. Time-varying Network Topology

In this paper, we assumed that the network topology and the probability of successful transmissions are time-invariant. The generalization to the case of time-varying network topology and transmission probabilities is straight forward. We define a new variable referred to as *network topology state* which determines the quality of all the links of the network. Let $X(t)$ be a stationary random process denoting the network topology state at time t. Suppose $X(t)$ take values from the space $\mathfrak{X}$ with probability distribution $P(X(t) = x) = \pi(x)$, for all $x \in \mathfrak{X}$.

Let $P_x(S|i)$, $S \subseteq \Omega$, $i \in \Omega$, be the local broadcast model of the network when the topology state is x. The behavior of the wireless channel can be characterized as the average behavior in different topology states using $\hat{P}(S|i)$, $S \subseteq \Omega$, $i \in \Omega$, i.e.

$$\hat{P}(S|i) = \sum_{x \in X} \pi(x) P_x(S|i). \tag{12}$$

In this new setting, ORCD can be defined using a fixed point equation as in (11) by replacing $P(S|i)$ with $\hat{P}(S|i)$. This generalization maintains the throughput optimality of ORCD.

C. Interference and Scheduling

So far we have assumed that the nodes transmit over orthogonal channels and hence, there is no inter-channel interference in the network. This assumption allowed for a clear presentation of the routing problem and illumination of the main concepts in their simplest forms. However, again, the generalization to the networks with inter-channel interference follows directly from [6]. The price of this generalization is shown to be the centralization of the routing/scheduling globally across the network or a constant factor performance loss of the distributed variants [6]. In a similar fashion to [10], [11], optimal power control algorithms or scheduling can be accommodated.

D. Imperfect Control Information

In this paper we assumed that ACK and FO packets are not subject to error or delay. In practice, this assumption is a good one only 1) if there exists a reliable method of communication for sending control packets (lower transmission rate, higher transmission power, etc.); or 2) if the reliability of the channel remains consistent for the duration of the routing decision cycle.

In cases of loss of ACK due to an interference or low SNR, the incorrect knowledge about the potential forwarders will be used in routing which potentially can result in unbounded congestion. Loss of FO packets, however, can result in the drop of data packets at all of the potential forwarders, hence, reducing the rate of packet delivery to the destination.

VI. Discussion and Future Work

In this paper, combining the important aspects of shortest path routing with those of backpressure routing, we provided an opportunistic routing policy with congestion diversity (ORCD) in which the nodes route packets according to a rank ordering of the nodes based on a congestion cost measure. We also introduced two practical modifications of ORCD referred to as Infreq-ORCD and AD-ORCD. Infreq-ORCD allows the use of outdated backlog information in its computation of the rank ordering of the nodes; while AD-ORCD allows for an asynchronous distributed computation. While we established the throughput optimality of ORCD and Infreq-ORCD, throughput optimality of AD-ORCD remains as a future area of study.

Furthermore, in this paper, we considered a single destination scenario. The generalization to the case of multi-destination scenario is believed to be straight forward and is currently a topic of study.

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Appendix

A. Sketch of the proof of Theorem 1

In this appendix we provide the sketch of the proof for the throughput optimality of ORCD for a network in which each node has a positive probability path to the destination.[3] See [16] for the details of the proof. The proof is based on the following corollary to *Foster-Lyapunov Theorem*.

Fact 1 (Lemma 4.1 in [19]). Let $L^* : \mathbb{R}_+^{\mathbb{N}} \to \mathbb{R}_+$ be a Lyapunov function. If there exist constants $B > 0$, $\epsilon > 0$, such that for all time slots t we have:

$$\mathbb{E}\left[L^*(\boldsymbol{Q}(t+1)) - L^*(\boldsymbol{Q}(t)) | \boldsymbol{Q}(t)\right] \leq B - \epsilon \sum_{k=1}^{N} Q_k(t), \quad (13)$$

then the network is stable.

To prove Theorem 1, we identify a Lyapunov function satisfying (13) for any arrival rate vector $\boldsymbol{\lambda} \in int(\mathfrak{S})$. The following definitions are required.

Definition. A *rank ordering* $R = (C_1, C_2, \ldots, C_M)$ is an ordered list of non-empty sets $C_1, C_2, \ldots, C_M$ $(1 \leq M \leq N)$, referred to as *ranking classes*, that make up a partition of $\{1, 2, \ldots, N\}$, i.e. $\cup_{i=1}^{M} C_i = \{1, 2, \ldots, N\}$ and $C_i \cap C_j = \emptyset$, $i \neq j$.

Definition. A rank ordering $R = (C_1, C_2, \ldots, C_M)$ is referred to as *path-connected* if for each node $i \in C_k$, $1 \leq k \leq M$, there exist distinct nodes $j_1, j_2, \ldots, j_l \in C^{k-1}$ such that $p_{ij_1} > 0, p_{j_1 j_2} > 0, \ldots, p_{j_l 0} > 0$. The set of all path-connected rank orderings is denoted by $\mathcal{R}_c$.

Definition. Given two rank orderings $R = (C_1, C_2, \ldots, C_M)$ and $R' = (C'_1, C'_2, \ldots, C'_{M+1})$, we say R' is a *one-step refinement* of R (and R is a *one-step confinement* of R') with regard to ranking class C_i if

$$\begin{cases} C_k = C'_k & \text{if } 1 \leq k \leq i-1 \\ C_i = C'_i \cup C'_{i+1} & \\ C_k = C'_{k+1} & \text{if } i+1 \leq k \leq M \end{cases}.$$

We denote the set of all path-connected one-step refinements and confinements of R by $\mathcal{A}_c(R)$.

Definition. Let $R = (C_1, C_2, \ldots, C_M)$ and $R' = (C'_1, C'_2, \ldots, C'_{M'})$. We define a *mismatch* $m : \mathcal{R}_c \times \mathcal{R}_c \to \mathbb{N}$ as

$$m(R, R') = \min\left\{i \in \mathbb{N} : C_i \neq C'_i\right\}.$$

Let f be a bivariate function of the following form:

$$f(m, n) = \frac{1}{K^m (K^n - 1)} \quad \text{for all } m \geq 0, n > 0, \quad (14)$$

where $K = 1 + \frac{1}{p_{min}}$ for $p_{min} = \min\{p_{ij} : i, j \in \Omega, p_{ij} > 0\}$.

Definition. Given f, a *penalty* function Λ_f is defined on backlog vector $\boldsymbol{Q} \in \mathbb{R}_+^N$, rank ordering $R = (C_1, C_2, \ldots, C_M) \in \mathcal{R}_c$, and natural number n, $n \leq M$, as follows

$$\Lambda_f(\boldsymbol{Q}, R, n) = \sum_{i=1}^{n} f(|C^{i-1}|, |C_i|) Q_{C_i}.$$

[3] If a node has no path to the destination, it cannot sustain any traffic and can be ignored without loss of generality.

Definition. Consider rank orderings R and $R' \in \mathcal{A}_c$. We say *R penalizes $\boldsymbol{Q}$ less than R'* and write $R <_{\boldsymbol{Q}} R'$ if

- $\Lambda_f(\boldsymbol{Q}, R, m(R, R')) < \Lambda_f(\boldsymbol{Q}, R', m(R, R'))$, or if
- $\Lambda_f(\boldsymbol{Q}, R, m(R, R')) = \Lambda_f(\boldsymbol{Q}, R', m(R, R'))$ and R is a one-step refinement of R'.

We define $D_f^c(R)$, $R \in \mathcal{R}_c$, as

$$D_f^c(R) = \left\{\boldsymbol{Q} \in \mathbb{R}_+^N : R <_{\boldsymbol{Q}} R' \text{ for all } R' \in \mathcal{A}_c(R)\right\}. \quad (15)$$

Remark Let R and R' be two rank orderings and let $\eta \in \mathbb{R}^+$ be a constant. If $R <_{\boldsymbol{Q}} R'$ then $R <_{\eta \boldsymbol{Q}} R'$. In other words, $D_f^c(R)$ is a cone in $\mathbb{R}_+^N$.

Next lemma renders the set of cones as a partition of $\mathbb{R}_+^N$.

Lemma 1. *For all $\boldsymbol{Q} \in \mathbb{R}_+^N$, there exists a unique $R \in \mathcal{R}_c$ such that $\boldsymbol{Q} \in D_f^c(R)$.*

We construct a piece-wise Lyapunov function, $L_f^* : \mathbb{R}_+^N \to \mathbb{R}_+$, by assigning to each cone $D_f^c(R)$, $R = (C_1, C_2, \ldots, C_M) \in \mathcal{R}_c$, a weighted quadratic function of the form:

$$L_f(\boldsymbol{Q}, R) = \sum_{i=1}^{M} f(|C^{i-1}|, |C_i|) Q_{C_i}^2. \quad (16)$$

Since the collection of cones form a partition of $\mathbb{R}_+^N$, we can combine the above quadratic functions to arrive at a piece-wise quadratic function

$$L_f^*(\boldsymbol{Q}) = \sum_{R \in \mathcal{R}_c} L_f(\boldsymbol{Q}, R) \mathbf{1}_{\{\boldsymbol{Q} \in D_f^c(R)\}}. \quad (17)$$

Lemma 2. *$L^*(\cdot)$ is a continuous and differentiable Lyapunov function.*

Example 1. Consider a network of four nodes as given in Fig. 5(a). Note that $(\{2\}, \{1\}, \{3\})$, $(\{2\}, \{3\}, \{1\})$, and $(\{2\}, \{1, 3\})$, are not path-connected. Fig. 5(b) shows the structure of the cones $\{D_f^c(R)\}_{R \in \mathcal{R}_c}$ and also the corresponding functions $\{L_f(\cdot, R)\}_{R \in \mathcal{R}_c}$ where $\mathcal{R}_c$ is the set of all path-connected rank orderings of $\{1, 2, 3\}$.

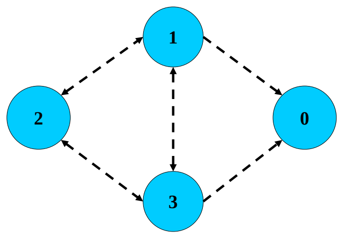
Since the collection of cones $\{D_f^c(R)\}_{R \in \mathcal{R}_c}$ partitions $\mathbb{R}_+^N$, it is meaningful to define function $U_f : \Omega \times \mathbb{R}_+^N \to \mathbb{R}_+$ such that

$$U_f(k, \boldsymbol{Q}) = f(|C^{i-1}|, |C_i|) Q_{C_i}(t), \quad (18)$$

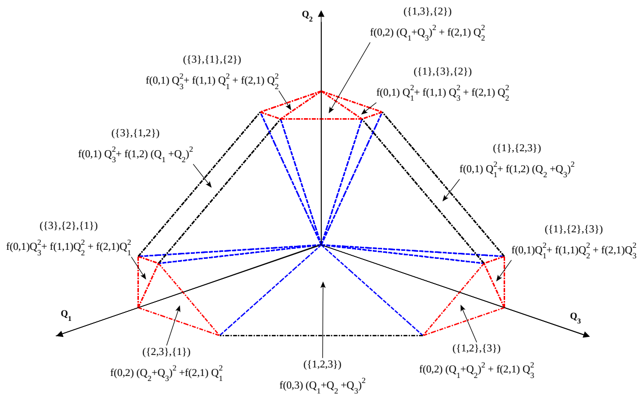
where $\boldsymbol{Q} \in D_f^c(R)$, $R = (C_1, C_2, \ldots, C_M)$, and $k \in C_i$.

Next we provide the main steps in showing L^* has a negative expected drift.

Let us consider the Lyapunov drift when $\boldsymbol{Q}(t) \in D_f^c(R)$ for some $R = (C_1, C_2, \ldots, C_M) \in \mathcal{R}_c$. For ease of notation and exposition define $A_C(t) = \sum_{i \in C} A_i(t)$, $Q_C(t) = \sum_{i \in C} Q_i(t)$, $\mu_{C,in}(t) = \sum_{j \notin C} \sum_{k \in C} \mu_{jk}(t)$, and $\mu_{C,out}(t) =$



(a) A network of four nodes



(b) Structure of the cones and the Lyapunov function

Fig. 5. Structure of the path-connected cones and the Lyapunov function for a network of four nodes

$\sum_{j\in C}\sum_{k\notin C}\mu_{jk}(t)$. In [16], it is shown that

$$\begin{aligned}&\mathbb{E}\left[L_f^*(\boldsymbol{Q}(t+1))-L_f^*(\boldsymbol{Q}(t))|\boldsymbol{Q}(t)\right]\\&\le B_f-2\sum_{i=1}^{M}f(|C^{i-1}|,|C_i|)Q_{C_i}(t)\mathbb{E}[\mu^*_{C_i,out}(t)-\\&\quad\mu^*_{C_i,in}(t)-A_{C_i}(t)|\boldsymbol{Q}(t)]+o(\|\boldsymbol{Q}(t+1)-\boldsymbol{Q}(t)\|),\end{aligned}\tag{19}$$

where B_f is a constant bounded real number and $\{\mu^*_{ij}(t)\}_{i,j\in\Omega}$ represent the routing decisions under ORCD.

Lemma 3. *Routing decisions under ORCD are such that* $\mu^*_{ij}=1$, *only when* $j\in S_i(t)$ *and* $U_f(j,\boldsymbol{Q}(t))\le U_f(k,\boldsymbol{Q}(t))$ *for all* $k\in S_i(t)$.

Hence, for any collection of routing decisions $\{\mu_{ij}(t)\}_{i,j\in\Omega}$, we have

$$\begin{aligned}&\sum_{i=1}^{M}f(|C^{i-1}|,|C_i|)Q_{C_i}(t)(\mu^*_{C_i,out}(t)-\mu^*_{C_i,in}(t))\\&\ge\sum_{i=1}^{M}f(|C^{i-1}|,|C_i|)Q_{C_i}(t)(\mu_{C_i,out}(t)-\mu_{C_i,in}(t)).\end{aligned}\tag{20}$$

Fact 2 (Corollary 1 in [6]). An arrival rate vector $\boldsymbol{\lambda}$ is within the stability region $\mathfrak{S}$ if and only if there exists a stationary randomized routing policy that makes routing decisions $\{\tilde{\mu}_{ij}(t)\}_{i,j\in\Omega}$, solely based on the collection of potential forwarders at time t, $\{S_i(t)\}_{i\in\Omega}$, and for which

$$\mathbb{E}\left[\sum_{j\in\Omega}\tilde{\mu}_{kj}(t)-\sum_{i\in\Omega}\tilde{\mu}_{ik}(t)\right]\ge\lambda_k.$$

However, since $\boldsymbol{\lambda}\in int(\mathfrak{S})$, there exists a positive vector $\boldsymbol{\epsilon}$ (vector of length N with all elements equal to ϵ, $\epsilon>0$) such that $\boldsymbol{\lambda}+\boldsymbol{\epsilon}\in\mathfrak{S}$. In other words, from Fact 2

$$\mathbb{E}\left[\tilde{\mu}_{C_i,out}(t)-\tilde{\mu}_{C_i,in}(t)-A_{C_i}(t)|\boldsymbol{Q}(t)\right]\ge\epsilon.\tag{21}$$

Combining (20) and (21) with (19), we obtain the following upper bound of the Lyapunov drift when packets are routed according to ORCD:

$$\begin{aligned}&\mathbb{E}\left[L_f^*(\boldsymbol{Q}(t+1))-L_f^*(\boldsymbol{Q}(t))|\boldsymbol{Q}(t)\right]\\&\le B_f-2\epsilon\sum_{i=1}^{M}f(|C^{i-1}|,|C_i|)Q_{C_i}(t)+o(\|\boldsymbol{Q}(t+1)-\boldsymbol{Q}(t)\|).\end{aligned}\tag{22}$$

Now the assertion of the theorem follows from

$$f(0,|C_1|)\ge f(|C^1|,|C_2|)\ge\cdots\ge f(|C^{M-1}|,|C_M|)\ge f(N,1),\tag{23}$$

and by letting $B'_f=B_f+o(\|\boldsymbol{Q}(t+1)-\boldsymbol{Q}(t)\|)$ and $\epsilon'=2\epsilon f(N,1)$.

B. Proof of Theorem 2

In this appendix, we provide the proof for the throughput optimality of Infreq-ORCD. The proof is again based on Fact 1 and we use the same Lyapunov function as in the proof of Theorem 1.

Let $\{\hat{\mu}_{ij}(t)\}_{i,j\in\Omega}$ represent the routing decisions under Infreq-ORCD. Similar to (19), we can find an upper bound on the Lyapunov drift when $\boldsymbol{Q}(t)\in D_f^c(R)$ for some $R=(C_1,C_2,\ldots,C_M)\in\mathcal{R}_c$, and under Infreq-ORCD:

$$\begin{aligned}&\mathbb{E}\left[L_f^*(\boldsymbol{Q}(t+1))-L_f^*(\boldsymbol{Q}(t))|\boldsymbol{Q}(t)\right]\\&\le B_f-2\sum_{i=1}^{M}f(|C^{i-1}|,|C_i|)Q_{C_i}(t)\mathbb{E}[\hat{\mu}_{C_i,out}(t)-\\&\quad\hat{\mu}_{C_i,in}(t)-A_{C_i}(t)|\boldsymbol{Q}(t)]+o(\|\boldsymbol{Q}(t+1)-\boldsymbol{Q}(t)\|),\end{aligned}\tag{24}$$

where B_f is a constant bounded real number.

Claim 1. Let $\hat{\boldsymbol{Q}}(t)$ denote the outdated backlog used by Infreq-ORCD. We have

$$\begin{aligned}&-2\sum_{i=1}^{M}f(|C^{i-1}|,|C_i|)Q_{C_i}(t)(\hat{\mu}_{C_i,out}(t)-\hat{\mu}_{C_i,in}(t))\\&\le-2\sum_{i=1}^{M}f(|C^{i-1}|,|C_i|)Q_{C_i}(t)(\mu^*_{C_i,out}(t)-\mu^*_{C_i,in}(t))\\&\quad+o(\|\boldsymbol{Q}(t)-\hat{\boldsymbol{Q}}(t)\|),\end{aligned}\tag{25}$$

where $\{\mu^*_{ij}(t)\}_{i,j\in\Omega}$ and $\{\hat{\mu}_{ij}(t)\}_{i,j\in\Omega}$ represent the routing decisions under ORCD and Infreq-ORCD respectively.

Combining (24) and (25), we obtain

$$\begin{aligned}&\mathbb{E}\left[L_f^*(\boldsymbol{Q}(t+1))-L_f^*(\boldsymbol{Q}(t))|\boldsymbol{Q}(t)\right]\\&\leq \hat{B}_f-2\sum_{i=1}^{M}f(|C^{i-1}|,|C_i|)Q_{C_i}(t)\mathbb{E}[\mu^*_{C_i,out}(t)-\\&\quad\mu^*_{C_i,in}(t)-A_{C_i}(t)|\boldsymbol{Q}(t)]+o(\|\boldsymbol{Q}(t+1)-\boldsymbol{Q}(t)\|),\quad(26)\end{aligned}$$

where $\hat{B}_f=B_f+o(\|\boldsymbol{Q}(t)-\hat{\boldsymbol{Q}}(t)\|)$. Note that $\hat{B}_f$ is bounded due to the boundedness of arrivals $A(t)$ and T. The rest of the proof is similar to the proof of Theorem 1 and is omitted for brevity.

What remains is to prove Claim 1:

Suppose node i's transmission at time t is received by potential forwarders $S_i(t)$. Furthermore, suppose that nodes $a,b\in S_i(t)$ are the nodes with the highest rank under ORCD and Infreq-ORCD, respectively, i.e. $\mu^*_{ia}(t)=\hat{\mu}(t)_{ib}=1$. From Lemma 3 we have

$$\begin{aligned}U_f(a,\boldsymbol{Q}(t))&\leq U_f(b,\boldsymbol{Q}(t)),&(27)\\U_f(a,\hat{\boldsymbol{Q}}(t))&\geq U_f(b,\hat{\boldsymbol{Q}}(t)).&(28)\end{aligned}$$

In order to prove Claim 1 it suffices to show that

$$-\left(U_f(a,\boldsymbol{Q}(t))-U_f(b,\boldsymbol{Q}(t))\right)=o(\|\boldsymbol{Q}(t)-\hat{\boldsymbol{Q}}(t)\|).\quad(29)$$

Consider the line that connects $\boldsymbol{Q}(t)$ and $\hat{\boldsymbol{Q}}(t)$ in $\mathbb{R}^N_+$. Suppose this line goes through $M-1$ cones in $\mathbb{R}^N_+$. Let $Z_1,Z_2,\ldots,Z_M$ be respectively the intersection of the line connecting $\boldsymbol{Q}(t)$ to $\hat{\boldsymbol{Q}}(t)$ with the M separating hyperplanes of the $M-1$ cones between them, i.e.

$$\begin{aligned}&\|\boldsymbol{Q}(t)-\hat{\boldsymbol{Q}}(t)\|\\&=\|\boldsymbol{Q}(t)-\boldsymbol{Z_1}\|+\|\boldsymbol{Z_1}-\boldsymbol{Z_2}\|+\cdots+\|\boldsymbol{Z_M}-\hat{\boldsymbol{Q}}(t)\|.\quad(30)\end{aligned}$$

Note that since $Z_1,Z_2,\ldots,Z_M$ are on the hyperplanes, each two consecutive points in set $\{\boldsymbol{Q}(t),Z_1,Z_2,\ldots,Z_M,\hat{\boldsymbol{Q}}(t)\}$ can be considered to belong to the same cone, and hence, have same rank ordering of the nodes. From definition of function U_f, we obtain

$$\begin{aligned}|U_f(a,\boldsymbol{Q}(t))-U_f(a,\boldsymbol{Z_1})|&=o(\|\boldsymbol{Q}(t)-\boldsymbol{Z_1}\|),\\|U_f(a,\boldsymbol{Z_m})-U_f(a,\boldsymbol{Z_{m+1}})|&=o(\|\boldsymbol{Z_m}-\boldsymbol{Z_{m+1}}\|),\ 1\leq m\leq M,\\|U_f(a,\boldsymbol{Z_M})-U_f(a,\hat{\boldsymbol{Q}}(t))|&=o(\|\boldsymbol{Z_M}-\hat{\boldsymbol{Q}}(t)\|).\end{aligned}$$

Therefore,

$$\begin{aligned}&U_f(a,\boldsymbol{Q}(t))-U_f(a,\hat{\boldsymbol{Q}}(t))\\&=[U_f(a,\boldsymbol{Q}(t))-U_f(a,\boldsymbol{Z_1})]+[U_f(a,\boldsymbol{Z_1})-U_f(a,\boldsymbol{Z_2})]\\&\quad+\cdots+[U_f(a,\boldsymbol{Z_M})-U_f(a,\hat{\boldsymbol{Q}}(t))]=o(\|\boldsymbol{Q}(t)-\hat{\boldsymbol{Q}}(t)\|).\\&\hfill(31)\end{aligned}$$

We can derive the same result for all other nodes in the network. In other words, there exist constants η_a,η_b such that

$$\begin{aligned}U_f(a,\boldsymbol{Q}(t))&=U_f(a,\hat{\boldsymbol{Q}}(t))+\eta_a\|\boldsymbol{Q}(t)-\hat{\boldsymbol{Q}}(t)\|,&(32)\\U_f(b,\boldsymbol{Q}(t))&=U_f(b,\hat{\boldsymbol{Q}}(t))+\eta_b\|\boldsymbol{Q}(t)-\hat{\boldsymbol{Q}}(t)\|.&(33)\end{aligned}$$

However, (27), (28), (32), and (33) imply that $\eta_a\leq\eta_b$ and

$$-\left(U_f(a,\boldsymbol{Q}(t))-U_f(b,\boldsymbol{Q}(t))\right)\leq(\eta_b-\eta_a)\|\boldsymbol{Q}(t)-\hat{\boldsymbol{Q}}(t)\|.$$

With this, the proof is now complete.

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