1. Plot poisson distribution for $\lambda = 1$, $\lambda = 4$, $\lambda = 10$ using Matlab.

2. Expected distribution. What is the expected vertex degree distribution for an Erdos-Renyi graph with 100 nodes and edge probability $p = 0.01$. Call this value $\lambda$. Plot the node degree distribution for this value of $\lambda$.

3. In class we encountered the Pareto distribution, and the exponential distribution. In this problem we will visually understand why the former is said to be a heavy-tail distribution.

   A continuous random variable has the Pareto distribution with parameters $\alpha > 0$ and $x_m > 0$ if its probability density function is given by
   \[
   f_P(x; \alpha, x_m) = \begin{cases} \frac{x_m^\alpha}{x^\alpha} & \text{for } x \geq x_m, \\ 0 & \text{for } x < x_m. \end{cases}
   \]

   The mean is given by
   \[
   m_P(\alpha, x_m) = \begin{cases} \frac{x_m\alpha}{\alpha-1} & \text{for } \alpha > 1, \\ \infty & \text{for } \alpha \leq 1. \end{cases}
   \]

   A continuous random variable has the Exponential distribution with parameter $\lambda > 0$ if its probability density function is given by
   \[
   f_E(x; \lambda) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0, \\ 0 & \text{for } x < 0. \end{cases}
   \]

   The mean is given by $m_E(\lambda) = 1/\lambda$.

1) Fix a value of $\alpha$ of your choice, and (using the programming language of your choice) plot $f_E(x; \lambda)$ in log-log scale.

2) For $\alpha \in \{1, 2, 3, 4, 5, 6, 7, 8\}$, consider the distribution $f_P(x; \alpha, x_m)$ such that $m_P(\alpha, x_m) = m_E(\lambda)$. That is, for each $\alpha$ you have to set $x_m\alpha = 1/\lambda$, and compute the corresponding value of $x_m$ (call it $x_m(\alpha)$).

3) For $\alpha \in \{1, 2, 3, 4, 5, 6, 7, 8\}$, plot $f_P(x; \alpha, x_m(\alpha))$, for the value of $x_m(\alpha)$ computed in part 2), in the same figure were you plotted $f_E(x; \lambda)$. You should now have an understating why the Pareto distribution is said to have an heavy tail. Explain your understanding.

4. Relationship between Poisson and exponential distributions.

   Let $N$ be the no. of packets arriving at a switch during the time interval $(0, t)$ with a Poisson degree distribution, i.e., $N \sim \text{Poisson} (\lambda t)$. Let $X$ be the time until first packet arrival. Find the distribution of $X$. 