ECE 45 Homework 1

Question 1:

A. Write in the form $a+jb$:
   (a) $e^{-j\pi/4}$
   (b) $e^{1+j}$
   (c) $e^{3+j}$

B. Compute the following for $X = 1 + e^{-j\pi/3} + e^{j\pi/3} + e^{j\pi}$
   (a) Real part of $X$
   (b) Phase of the complex conjugate of $X$
   (c) Express $X/Y$ in polar form where $Y=1+j$

Question 2:

A series RLC circuit with $L=160\,\text{mH}$, $C=100\,\text{uF}$, $R=40\,\text{ohm}$ is connected to a sinusoidal voltage $V(t)=(40\,\text{V})\sin(200t)$.

(a) What is the impedance of the circuit?
(b) Let the current at any instant in the circuit be $I(t)=I_0\sin(\omega t-\theta)$. Find $I_0$.
(c) What is the phase constant $\theta$?

Question 3:

Suppose a sinusoidal voltage source $V(t)=(150\,\text{V})\sin(100t)$ is connected to a series RLC circuit with $R=40\,\text{ohms}$, $L=80\,\text{mH}$ and $C=50\,\text{uF}$ as shown in the figure.

(a) Calculate $V_{R_0}$, $V_{L_0}$ and $V_{C_0}$ which are the maximum value of the voltage drops across these elements.
(b) Calculate the maximum potential difference across the inductor and capacitor between points b and d shown in the figure.

Question 4:

A RL high pass filter (circuit that filters out low frequency AC currents) can be represented by the circuit in the figure shown below, where $R$ is the internal resistance of the inductor.
(a) Find $V_{20}/V_{10}$, the ratio of the maximum output voltage $V_{20}$ to the maximum input voltage $V_{10}$.

(b) Suppose $r=15\text{ohms}$, $R=10\text{ohms}$ and $L=250\text{mH}$. Find the frequency at which $V_{20}/V_{10}=1/2$.

Question 5:
Consider the circuit shown in the figure below:

(a) Compute the transfer function $V_{\text{out}}/V_{\text{in}}$.

(b) Sketch the Bode magnitude and phase plot for the above transfer function. Take $R=1\text{kohms}$ and $C=1\text{uF}$.

(c) For $V_{\text{in}}=2\cos(t)+2\cos(100000t)$ use the bode magnitude and phase plots to find the output $V_{\text{out}}(t)$. Verify your answer using the phasor diagram analysis covered in class. (Hint: Notice that the input is a sum of low frequency input and a high frequency input. Analyze the circuit for the limiting case of $\omega \to 0$ and $\omega \to \infty$).

Question 6:
Sketch the Bode magnitude and phase plots for the following transfer function:

$$H(j\omega) = \frac{(10^{10}(j\omega)^2 + 10j\omega))}{(j\omega + 100)^2 (j\omega + 10000)}$$

Question 7:
Consider the circuit shown below with $v(t) = 10+\sin(4t)+\cos(8t)$. Determine the frequency response and the output $V_{out}(t)$.

Question 8:

Consider a series RLC circuit with $R=50\, \text{ohm}$, $L=10\, \text{nH}$ and $C=1\, \text{uF}$. Use MATLAB to do the following:

(a) Plot the magnitude and phase of the total impedance as a function of frequency.
(b) Determine the frequency at which the impedance seen is purely resistive. Use several L and C values and notice how this frequency shifts. This frequency is known as the resonant frequency.

Consider the circuit below:

(a) Use MATLAB to evaluate the transfer function from the input to the output i.e. the frequency response.
(b) Use MATLAB to sketch Bode magnitude and phase plots for the same.