Problem 1. The signal $y(t)$ is generated by convolving a band-limited signal $x_1(t)$ with another band-limited signal $x_2(t)$, that is,

$$y(t) = x_1(t) * x_2(t)$$

where,

$$X_1(\omega) = 0 \quad for \quad |\omega| > 1000\pi$$
$$X_2(\omega) = 0 \quad for \quad |\omega| > 2000\pi$$

Impulse train sampling is performed on $y(t)$ to obtain

$$y_p(t) = \sum_{n=-\infty}^{+\infty} y(nT) \delta(t - nT)$$

Specify the range of values for the sampling period $T$ which ensures that $y(t)$ is recoverable from $y_p(t)$.

Problem 2. The sampling theorem, as we have derived it, states that a signal $x(t)$ must be sampled at a rate higher than its bandwidth (or equivalently, a rate greater than its highest frequency). This implies that if $x(t)$ has a spectrum as indicated in Figure 2.a then $x(t)$ must be sampled at a rate greater than $2\omega_2$. However, since the signal has most of its energy concentrated in a narrow band, it would seem reasonable to expect that a sampling rate lower than twice the highest frequency could be used.

When $x(t)$ is real, a procedure for “bandpass” sampling and reconstruction is as outlined in this problem. It consists of multiplying $x(t)$ by a complex-exponential and then sampling the product. The sampling system is shown in Figure 2.b. With $x(t)$ real and with $X(\omega)$ nonzero only for $\omega_1 < |\omega| < \omega_2$, the frequency is chosen to be $\omega_0 = (\omega_1 + \omega_2)$, and the lowpass filter $H_1(\omega)$ has cutoff frequency $\left(\frac{1}{2}\right)(\omega_2 - \omega_1)$.

(a) For $X(\omega)$ as shown in Figure 2.a, sketch $X_p(\omega)$.

(b) Determine the maximum sampling period $T$ such that $x(t)$ is recoverable from $x_p(t)$.

(c) Determine a system to recover $x(t)$ from $x_p(t)$.
Problem 3. The frequency which, under the sampling theorem, must be exceeded by the sampling frequency is called the Nyquist rate. Determine the Nyquist rate corresponding to each of the following signals:

(a) $3 \cos 50\pi t + 10 \sin 300\pi t - \cos 100\pi t$
(b) $\text{sinc}(200t)$
(c) $\text{sinc}^2(200t)$
(d) $\text{sinc}(200t) + \text{sinc}^2(200t)$

Note: $\text{sinc}(x) \equiv \frac{\sin \pi x}{\pi x}$

Problem 4. Let $x(t)$ be a signal with Nyquist rate $\omega_o$. Determine the Nyquist rate for each of the following signals:

(a) $x(t) + x(t - 1)$
(b) $\frac{dx(t)}{dt}$
(c) $x^2(t)$
(d) $x(t) \cos(\omega_o t)$

Problem 5. Compute the convolution of each of the following pairs of signals $x(t)$ and $h(t)$ by calculating $X(\omega)$ and $H(\omega)$, using the convolution property, and inverse transforming.

(a) $x(t) = te^{-2t}u(t)$, $h(t) = e^{-4t}u(t)$
(b) $x(t) = te^{-2t}u(t)$, $h(t) = te^{-4t}u(t)$
(c) $x(t) = e^{-t}u(t)$, $h(t) = e^t u(-t)$

Problem 6. For $x(t)$ and $h(t)$ as shown in Figure 6, find $x(t) * h(t)$.

![Figure 6](image)
Problem 7. For \( x(t) \) and \( h(t) \) as shown in Figure 7, find \( x(t) \cdot h(t) \).

![Figure 7](image-url)