1. Consider a network of size $N$ with $L$ links, where traveling over each link takes 1 unit of time. Furthermore, the transmission over a link $(i, j)$ has a cost $c_{ij} > 0$ in terms of energy cost. Consider a packet to be routed to node 1 in a maximum of $T$ units of time, where $T$ is a fixed and finite number.

   - Devise an algorithm that for each node $i$ finds the path of least total energy cost while guaranteeing to deliver the packet within $T$ units of time.

Now generalize the above: There are 3 types of packets to be delivered in the network: 1) urgent packets which are required to be received at the destination as fast as possible, 2) a class of time sensitive packets which are required to be delivered within $T$ unit of time and finally 3) a class of delay-tolerant packet.

   - How does this impact the question of routing in the network?

2. Consider the TCP Reno protocol discussed in the class. Figure 1 shows variations of the window size over time. Answer the following questions.

   (a) Determine intervals of time that TCP is operating in slow start and congestion avoidance (AIMD) phases.

   (b) What are the initial values of ssthresh and $W_{max}$ (maximum window size)?

   (c) What are the values of ssthresh at 15th and 30th transmission round respectively?

   (d) When is the 110th packet sent?

3. Consider a network with two users, five links, and $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$. Modify TCP Vegas algorithm such that $\alpha_1 = \alpha$ and $\alpha_2 = \frac{\alpha}{2}$ for some constant $\alpha$. Here $\alpha_i$ is where the target throughput is matched to the actual throughput (see page 61 of the book when $\alpha_i = \beta_i$, but assume a non-homogeneous case $\alpha_1 \neq \alpha_2$). Assume that propagation delay is the same for both users and equals $d_0$. All links have capacity $c$ pkts/ms.
Figure 1: Window size versus time under TCP Reno protocol.

(a) Show that when both users have perfect estimation of their propagation delay \( (d_0) \), the equilibrium point is weighted proportionally fair. Is the equilibrium point also max-min fair? Justify your answer.

(b) Now assume user 2 overestimates the propagation delay as \( d_0 + \epsilon \). Show that the equilibrium point will not be proportionally fair.

4. Consider vector \( p \) and scalar \( \alpha \neq 1 \). The notion of the proportional fairness can be generalized to \( (p, \alpha) \)-proportional fairness in the following sense: A vector of rates \( x \) is \( (p, \alpha) \)-proportionally fair if it is feasible \( (x^* \in \Delta) \) and for any other feasible vector \( x \)

\[
\sum_i p_i \frac{x_i - x_i^*}{(x_i^*)^\alpha} \leq 0.
\]

Show that a vector is \( (p, \alpha) \)-proportionally fair if and only if it solves the network utility problem for \( g(x) = \sum_i p_i f_\alpha(x_i) \) where \( f_\alpha(y) := \frac{x^{1-\alpha}}{(1-\alpha)} \).