Homework Set #6
Due: Thursday, May 24, 2012

1. For the (7, 4) Hamming code, calculate the probability of decoding to a wrong codeword if the code is used for a binary symmetric channel with bit error probability \( p \). Evaluate the formula you obtain for the following values of \( p \): \( p = 0.2, 0.1, 0.05, 0.01, 0.005 \), and \( 0.001 \).

2. Run a simulation that verifies the result found in Problem 1 for the case of \( p = 0.05 \). Do the following a sufficient number of times to get a good estimate of the decoded word error rate. Compare your simulation results with the analytic expression found in Problem 1.
   (a) Generate 4 random binary digits.
   (b) Encode them by calculating the 3 check digits.
   (c) Put the code word through a BSC with bit error probability \( p \).
   (d) Decode the received 7-bit word.
   (e) Compare the decoded word with the transmitted word and count word errors.

3. Repeat Problem 2 but now only count the errors in the information positions of the code words to obtain an estimate of the decoded bit error probability. Is the bit error probability larger or smaller than the word error probability?

4. Suppose the Hamming code is used to detect errors.
   (a) For \( i = 0, 1, 2, \ldots, 7 \), find the number of error patterns containing exactly \( i \) errors that the decoder will fail to detect?
   (b) If the code is used for a binary symmetric channel with bit error probability \( p \), find an analytic expression for the probability of an undetected word error as a function of \( p \).
   (c) Verify the result found in part (b) by simulation for \( p = 0.2 \).

5. Suppose the Hamming code is used to correct erasures.
   (a) For \( i = 0, 1, 2, \ldots, 7 \), find the number of erasure patterns containing exactly \( i \) erasures that the decoder will fail to correct?
(b) If the code is used for a binary erasure channel with bit erasure probability $p$, find an analytic expression for the probability of failing to correct the erasures.

(c) Verify the result found in part (b) by simulation for $p = 0.2$. 