Probability Space \((\mathcal{S}, \mathcal{F}, P)\)

- **Sample Space** \(\mathcal{S}\): the set of all possible outcomes.

- **Set of events** \(\mathcal{F}\): the set of subsets of \(\mathcal{S}\) that is a collection of events.

Event \(A \subseteq \mathcal{S}\) occurs if the outcome \(\omega\) of the random experiment is an element of \(A\) (i.e., \(\omega \in A\)).

- **Probability measure**, \(P\): A function on \(\mathcal{F}\) that assigns probabilities to events according to the axioms of probability.

\(\mathcal{F}\) (set of events) cannot be an arbitrary collection of subsets of \(\mathcal{S}\); it must "make sense".

\(\mathcal{F}\) must be a \(\sigma\)-algebra (\(\sigma\)-field), which is defined by the following a

1. \(\emptyset \in \mathcal{F}\)
2. If \(A \in \mathcal{F}\), then \(A^c \in \mathcal{F}\)
3. If \(A_1, A_2, A_3, \ldots \in \mathcal{F}\), then \(\bigcup_{i=1}^{\infty} A_i \in \mathcal{F}\)

Example:

1. \(\mathcal{S} = \{1, 2, 3, 4, 5, 6\}\).
   \(\mathcal{F} = 2^{\mathcal{S}} = \{\emptyset, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100\}\.

2. \(\mathcal{S} = \{1, 2, 3, 4, 5, 6\}\).
   \(\mathcal{F} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{1, 6\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{2, 6\}, \{3, 4\}, \{3, 5\}, \{3, 6\}, \{4, 5\}, \{4, 6\}, \{5, 6\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 2, 5\}, \{1, 2, 6\}, \{1, 3, 4\}, \{1, 3, 5\}, \{1, 3, 6\}, \{1, 4, 5\}, \{1, 4, 6\}, \{1, 5, 6\}, \{2, 3, 4\}, \{2, 3, 5\}, \{2, 3, 6\}, \{2, 4, 5\}, \{2, 4, 6\}, \{2, 5, 6\}, \{3, 4, 5\}, \{3, 4, 6\}, \{3, 5, 6\}, \{4, 5, 6\}, \{1, 2, 3, 4\}, \{1, 2, 3, 5\}, \{1, 2, 3, 6\}, \{1, 2, 4, 5\}, \{1, 2, 4, 6\}, \{1, 2, 5, 6\}, \{1, 3, 4, 5\}, \{1, 3, 4, 6\}, \{1, 3, 5, 6\}, \{1, 4, 5, 6\}, \{2, 3, 4, 5\}, \{2, 3, 4, 6\}, \{2, 3, 5, 6\}, \{2, 4, 5, 6\}, \{3, 4, 5, 6\}, \{1, 2, 3, 4, 5\}, \{1, 2, 3, 4, 6\}, \{1, 2, 3, 5, 6\}, \{1, 2, 4, 5, 6\}, \{1, 3, 4, 5, 6\}, \{2, 3, 4, 5, 6\}, \{1, 2, 3, 4, 5, 6\}\).

\(\mathcal{F}\) controls how coarsely/finely probabilities can be assigned.
(3) $\mathcal{B} = \mathbb{R} = (-\infty, \infty)$

$\mathcal{B}$ = Borel field = "the smallest $\sigma$-algebra that contains all open intervals $(a, b) = [a, b)$ or half-open & closed $(a, b] = (-\infty, a] \cup [b, \infty)$

(4) $\mathcal{L} = \mathbb{R}$

$\mathcal{G}$ = $\mathcal{F} = 2^\mathbb{R}$ = (too rich to assign probabilities under the standard axioms)