Lecture #4 Multiple access channels

(Reading: NIT 4.1, 4.2, 4.4, 4.5)

- Discrete memoryless multiple access channel
- Simple bounds on the capacity region
- Time sharing
- Capacity region
- Gaussian multiple access channel
- Extension to more than two senders

Multiple access communication system

- DM multiple access channel (MAC) \((\mathcal{X}_1 \times \mathcal{X}_2, p(y|x_1, x_2), \mathcal{Y})\)
- A \((2^{nR_1}, 2^{nR_2}, n)\) code:
  - Message sets: \([1 : 2^{nR_1}]\) and \([1 : 2^{nR_2}]\)
  - Encoder \(j = 1, 2\): \(x^n_j(m_j)\)
  - Decoder: \((\hat{m}_1(y^n), \hat{m}_2(y^n))\)
- \((M_1, M_2) \sim \text{Unif}([1 : 2^{nR_1}] \times [1 : 2^{nR_2}]): x^n_1(M_1)\) and \(x^n_2(M_2)\) independent
- Average probability of error: \(P_e(n) = P(\{\hat{M}_1, \hat{M}_2 \neq (M_1, M_2)\})\)
- \((R_1, R_2)\) achievable if \(\exists (2^{nR_1}, 2^{nR_2}, n)\) codes such that \(\lim_{n \to \infty} P_e(n) = 0\)
- Capacity region \(\mathcal{C}\): Closure of the set of achievable rate pairs \((R_1, R_2)\)
Simple bounds on the capacity region

- Maximum achievable individual rates:
  \[ C_1 = \max_{x_2, p(x_2)} I(X_1; Y | X_2 = x_2), \quad C_2 = \max_{x_1, p(x_1)} I(X_2; Y | X_1 = x_1) \]

- Upper bound on the sum-rate:
  \[ R_1 + R_2 \leq C_{12} = \max_{p(x_1)p(x_2)} I(X_1, X_2; Y) \]

Examples

- Binary multiplier MAC: \( X_1, X_2 \in \{0, 1\}, Y = X_1 \cdot X_2 \in \{0, 1\} \)

- Binary erasure MAC: \( X_1, X_2 \in \{0, 1\}, Y = X_1 + X_2 \in \{0, 1, 2\} \)
Time sharing

Proposition 4.1
If \((R'_1, R'_2), (R''_1, R''_2) \in \mathcal{C}\), then \((R_1, R_2) = (\alpha R'_1 + \bar{\alpha} R''_1, \alpha R'_2 + \bar{\alpha} R''_2) \in \mathcal{C}\) for \(\alpha \in [0, 1]\)

- The capacity region \(\mathcal{C}\) is convex

Proof: Time sharing argument
- Let \(\mathcal{C}'_k\) be a sequence of \((2^{kR'_1}, 2^{kR'_2}, k)\) codes with \(P^{(k)}_{e_1} \to 0\)
- Let \(\mathcal{C}''_k\) be a sequence of \((2^{kR''_1}, 2^{kR''_2}, k)\) codes with \(P^{(k)}_{e_2} \to 0\)
- Construct a new sequence of codes by using \(\mathcal{C}'_{an}\) for \(i \in [1 : an]\) and \(\mathcal{C}''_{\bar{\alpha}n}\) for \(i \in [an + 1 : n]\)

\[
\begin{array}{cc}
\alpha n & \bar{\alpha}n \\
\mathcal{C}'_{an} & \mathcal{C}''_{\bar{\alpha}n}
\end{array}
\]

- By the union of events bound, \(P^{(n)}_{e} \leq P^{(an)}_{e_1} + P^{(\bar{\alpha}n)}_{e_2} \to 0\)

Remarks:
- Time division is a special case of time sharing (between \((R_1, 0)\) and \((0, R_2))\)
- The capacity region of any (synchronous) channel is convex
\( \mathcal{R}(X_1, X_2) \) region

- For \((X_1, X_2) \sim p(x_1)p(x_2)\), let \( \mathcal{R}(X_1, X_2) \) be the set of \((R_1, R_2)\) such that
  \[
  R_1 \leq I(X_1; Y|X_2), \\
  R_2 \leq I(X_2; Y|X_1), \\
  R_1 + R_2 \leq I(X_1, X_2; Y)
  \]

- This region is always a pentagon since
  \[
  \max\{I(X_1; Y|X_2), I(X_2; Y|X_1)\} \leq I(X_1, X_2; Y) \leq I(X_1; Y|X_2) + I(X_2; Y|X_1)
  \]

Capacity region (Ahlsvede 1971, Liao 1972)

**Theorem 4.2.**
\( \mathcal{C} \) is the convex closure of the set \( \bigcup_{p(x_1)p(x_2)} \mathcal{R}(X_1, X_2) \)

- For binary erasure MAC example, outer bound is tight
Proof of achievability

- We show that $R(X_1, X_2)$ is achievable for every $(X_1, X_2) \sim p(x_1)p(x_2)$
- The rest of the proof follows by time sharing

- Codebook generation:
  - Independently generate $2^{nR_1}$ sequences $x_1^n(m_1) \sim \prod_{i=1}^n p_{X_1}(x_{1i}), m_1 \in [1 : 2^{nR_1}]$
  - Independently generate $2^{nR_2}$ sequences $x_2^n(m_2) \sim \prod_{i=1}^n p_{X_2}(x_{2i}), m_2 \in [1 : 2^{nR_2}]$

- Encoding:
  - To send message $m_1$, encoder 1 transmits $x_1^n(m_1)$
  - To send message $m_2$, encoder 2 transmits $x_2^n(m_2)$

- Decoding:
  - Successive cancellation decoding
  - Simultaneous decoding

Successive cancellation decoding

- Find unique $\hat{m}_1$ such that $(x_1^n(\hat{m}_1), y^n) \in T_n$
- If such $\hat{m}_1$ is found, find unique $\hat{m}_2$ such that $(x_1^n(\hat{m}_1), x_2^n(\hat{m}_2), y^n) \in T_{\epsilon}$
“Little” packing lemma

- Let \((X, Y) \sim p(x, y)\)
- Let \(\tilde{Y}^n \sim \prod_{i=1}^n p_Y(\tilde{y}_i)\)
- Let \(X^n(m) \sim \prod_{i=1}^n p_X(x_i), m \in \mathcal{A}, |\mathcal{A}| \leq 2^{nR}\), be pairwise independent of \(\tilde{Y}^n\)

There exists \(\delta(\varepsilon) \to 0\) as \(\varepsilon \to 0\) such that

\[
\lim_{n \to \infty} P\{ (X^n(m), \tilde{Y}^n) \in T_{\varepsilon}^{(n)} \text{ for some } m \in \mathcal{A} \} = 0, \quad \text{if } R < I(X; Y) - \delta(\varepsilon)
\]

Analysis of the probability of error

- Consider \(P(\mathcal{E})\) conditioned on \((M_1, M_2) = (1, 1)\)
- Error events:

\[
\mathcal{E}_1 = \{ (X^n(1), X^n_2(1), Y^n) \notin T_{\varepsilon}^{(n)} \},
\mathcal{E}_2 = \{ (X^n_1(m_1), Y^n) \in T_{\varepsilon}^{(n)} \text{ for some } m_1 \neq 1 \},
\mathcal{E}_3 = \{ (X^n_1(1), X^n_2(m_2), Y^n) \in T_{\varepsilon}^{(n)} \text{ for some } m_2 \neq 1 \}
\]

Thus, by the union of events bound

\[
P(\mathcal{E}) \leq P(\mathcal{E}_1) + P(\mathcal{E}_2) + P(\mathcal{E}_3)
\]

- By the LLN, \(P(\mathcal{E}_1) \to 0\)
- By the “little” packing lemma (\(|\mathcal{A}| = 2^{nR_1} - 1, X \leftarrow X_1\)),
  \(P(\mathcal{E}_2) \to 0\) if \(R_1 < I(X_1; Y) - \delta(\varepsilon)\)
- By the “little” packing lemma (\(|\mathcal{A}| = 2^{nR_2} - 1, X \leftarrow X_2, Y \leftarrow (X_1, Y)\)),
  \(P(\mathcal{E}_3) \to 0\) if \(R_2 < I(X_2; Y, X_1) - \delta(\varepsilon) = I(X_2; Y|X_1) - \delta(\varepsilon)\)
Simultaneous decoding

- Find unique $(\hat{m}_1, \hat{m}_2)$ such that $(x^n_1(\hat{m}_1), x^n_2(\hat{m}_2), y^n) \in T^{(n)}$

Analysis of the probability of error

- Consider $P(\mathcal{E})$ conditioned on $(M_1, M_2) = (1, 1)$

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<tr>
<th>$m_1$</th>
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- Error events:

$\mathcal{E}_1 = \{(X^n_1(1), X^n_2(1), Y^n) \notin T^{(n)}\}$,

$\mathcal{E}_2 = \{(X^n_1(m_1), X^n_2(1), Y^n) \in T^{(n)}$ for some $m_1 \neq 1\}$,

$\mathcal{E}_3 = \{(X^n_1(1), X^n_2(m_2), Y^n) \in T^{(n)}$ for some $m_2 \neq 1\}$,

$\mathcal{E}_4 = \{(X^n_1(m_1), X^n_2(m_2), Y^n) \in T^{(n)}$ for some $m_1 \neq 1, m_2 \neq 1\}$
Analysis of the probability of error

- Error events:
  \[ E_1 = \{(X_1^n(1), X_2^n(1), Y^n) \notin T(n)\} , \]
  \[ E_2 = \{(X_1^n(m_1), X_2^n(1), Y^n) \in T(n) \text{ for some } m_1 \neq 1\} , \]
  \[ E_3 = \{(X_1^n(1), X_2^n(m_2), Y^n) \in T(n) \text{ for some } m_2 \neq 1\} , \]
  \[ E_4 = \{(X_1^n(m_1), X_2^n(m_2), Y^n) \in T(n) \text{ for some } m_1 \neq 1, m_2 \neq 1\} \]

- By the LLN, \( P(E_1) \to 0 \)

- By the “little” packing lemma,
  \[
  P(E_2) \to 0 \text{ if } R_1 < I(X_1; Y|X_2) - \delta(\epsilon), \\
  P(E_3) \to 0 \text{ if } R_2 < I(X_2; Y|X_1) - \delta(\epsilon), \\
  P(E_4) \to 0 \text{ if } R_1 + R_2 < I(X_1, X_2; Y) - \delta(\epsilon)
  \]

Proof of the converse (Slepian–Wolf 1973)

- Show that: For any sequence of \((2^{nR_1}, 2^{nR_2}, n)\) codes with \( P_e^{(n)} \to 0, (R_1, R_2) \in \mathcal{C} \)

- Code induces empirical pmf
  \[
  (M_1, M_2, X_1^n, X_2^n, Y^n) \sim 2^{-n(R_1 + R_2)} p(x_1^n|m_1)p(x_2^n|m_2) \prod_{i=1}^n p_{Y|X_1,X_2}(y_i|x_{1i}, x_{2i})
  \]

- By Fano’s inequality,
  \[
  H(M_1, M_2 | Y^n) \leq n(R_1 + R_2) P_e^{(n)} + 1 = n \epsilon_n
  \]

- Bounds on rates:
  \[
  R_1 \leq \frac{1}{n} \sum_{i=1}^n I(X_{1i}; Y_i | X_{2i}) + \epsilon_n, \\
  R_2 \leq \frac{1}{n} \sum_{i=1}^n I(X_{2i}; Y_i | X_{1i}) + \epsilon_n, \\
  R_1 + R_2 \leq \frac{1}{n} \sum_{i=1}^n I(X_{1i}, X_{2i}; Y_i) + \epsilon_n
  \]
Proof of the converse

- **Time-sharing random variable**: \( Q \sim \text{Unif}[1:n] \) be independent of \((X^n_1, X^n_2, Y^n)\)
- Define \( X_1 = X_{1Q}, X_2 = X_{2Q}, Y = Y_Q \), then \( p(y_q | x_{1q}, x_{2q}) = p_{Y|X_1, X_2}(y_q | x_{1q}, x_{2q}) \)
- The bounds can be expressed as
  \[
  R_1 \leq I(X_1; Y|X_2, Q) + \epsilon_n, \\
  R_2 \leq I(X_2; Y|X_1, Q) + \epsilon_n, \\
  R_1 + R_2 \leq I(X_1, X_2; Y|Q)
  \]
  for some joint pmf \( p(x_1|q)p(x_2|q) \)
- Thus \((R_1, R_2)\) must be in the **closure** of the set of \((R_1, R_2)\) such that
  \[
  R_1 \leq I(X_1; Y|X_2, Q), \\
  R_2 \leq I(X_2; Y|X_1, Q), \\
  R_1 + R_2 \leq I(X_1, X_2; Y|Q)
  \]
  for some joint pmf \( p(q)p(x_1|q)p(x_2|q) \)
- Denote this region by \( C' \)

Proof of the converse

- Clearly the capacity region \( C \subseteq C' \)
- Can show that \( C' \subseteq C \)
- But neither \( C \) nor \( C' \) seems **computable**—cardinality of \( Q \)?
- Can show: \(|Q| \leq 2\); obtain the equivalent characterization of the capacity region

**Theorem 4.3**
The capacity region \( C \) is the set of \((R_1, R_2)\) such that
  \[
  R_1 \leq I(X_1; Y|X_2, Q), \\
  R_2 \leq I(X_2; Y|X_1, Q), \\
  R_1 + R_2 \leq I(X_1, X_2; Y|Q)
  \]
  for some pmf \( p(q)p(x_1|q)p(x_2|q) \) with \(|Q| \leq 2\)
- Can we achieve the region in Theorem 4.3 directly?
Proof of achievability of the alternative characterization

- Key idea: Coded time sharing (Han–Kobayashi 1981)
- Codebook generation:
  - Fix $p(q)p(x_1|q)p(x_2|q)$
  - Randomly generate a time-sharing sequence $q^n \sim \prod_{i=1}^n P_Q(q_i)$
  - Conditionally independently generate $x_1^n(m_1) \sim \prod_{i=1}^n P_{X_1|Q}(x_{1i}|q_i)$, $m_1 \in \{1 : 2^{nR_1}\}$
  - Conditionally independently generate $x_2^n(m_2) \sim \prod_{i=1}^n P_{X_2|Q}(x_{2i}|q_i)$, $m_2 \in \{1 : 2^{nR_2}\}$
- Encoding:
  - To send message $m_1$, encoder 1 transmits $x_1^n(m_1)$
  - To send message $m_2$, encoder 2 transmits $x_2^n(m_2)$
- Decoding:
  - Find the unique $(\hat{m}_1, \hat{m}_2)$ such that $(q^n, x_1^n(\hat{m}_1), x_2^n(\hat{m}_2), y^n) \in \mathcal{T}_e^{(n)}$

Analysis of the probability of error

- Consider $P(E)$ conditioned on $(M_1, M_2) = (1, 1)$

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- Error events:
  \[
  \mathcal{E}_1 = \{(Q^n, X_1^n(1), X_2^n(1), Y^n) \notin \mathcal{T}_e^{(n)}\},
  \]
  \[
  \mathcal{E}_2 = \{(Q^n, X_1^n(m_1), X_2^n(1), Y^n) \in \mathcal{T}_e^{(n)}\} \text{ for some } m_1 \neq 1,
  \]
  \[
  \mathcal{E}_3 = \{(Q^n, X_1^n(1), X_2^n(m_2), Y^n) \in \mathcal{T}_e^{(n)}\} \text{ for some } m_2 \neq 1,
  \]
  \[
  \mathcal{E}_4 = \{(Q^n, X_1^n(m_1), X_2^n(m_2), Y^n) \in \mathcal{T}_e^{(n)}\} \text{ for some } m_1 \neq 1, m_2 \neq 1
  \]
Packing lemma

- Let \((U, X, Y) \sim p(u, x, y)\)
- Let \((\tilde{U}^n, \tilde{Y}^n) \sim p(\tilde{u}^n, \tilde{y}^n)\) be arbitrarily distributed
- Let \(X^n(m) \sim \prod_{i=1}^n p_{X|U}(x_i|\tilde{u}_i), m \in \mathcal{A}, |\mathcal{A}| \leq 2^{nR}\),
  be pairwise conditionally independent of \(\tilde{Y}^n\) given \(\tilde{U}^n\)

There exists \(\delta(\epsilon) \to 0\) as \(\epsilon \to 0\) such that

\[
\lim_{n \to \infty} P\{(\tilde{U}^n, X^n(m), \tilde{Y}^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m \in \mathcal{A}\} = 0,
\]

if \(R < I(X; Y|U) - \delta(\epsilon)\)

- Generalizes “little” packing lemma in two says:
  - \(\tilde{Y}^n\) conditionally independent of \(X^n(m)\) given \(\tilde{U}^n\)
  - \((\tilde{U}^n, \tilde{Y}^n)\) can have an arbitrary pmf

Analysis of the probability of error

- Error events:
  \[
  \mathcal{E}_1 = \{(Q^n, X^n_1(1), X^n_2(1), Y^n) \notin \mathcal{T}_\epsilon^{(n)}\},
  \]
  \[
  \mathcal{E}_2 = \{(Q^n, X^n_1(m_1), X^n_2(1), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \neq 1\},
  \]
  \[
  \mathcal{E}_3 = \{(Q^n, X^n_1(1), X^n_2(m_2), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_2 \neq 1\},
  \]
  \[
  \mathcal{E}_4 = \{(Q^n, X^n_1(m_1), X^n_2(m_2), Y^n) \in \mathcal{T}_\epsilon^{(n)} \text{ for some } m_1 \neq 1, m_2 \neq 1\}
  \]
- By the LLN, \(P(\mathcal{E}_1) \to 0\)
- By the packing lemma,
  \[
P(\mathcal{E}_2) \to 0 \text{ if } R_1 < I(X_1; Y|X_2, Q) - \delta(\epsilon),
  \]
  \[
P(\mathcal{E}_3) \to 0 \text{ if } R_2 < I(X_2; Y|X_1, Q) - \delta(\epsilon),
  \]
  \[
P(\mathcal{E}_4) \to 0 \text{ if } R_1 + R_2 < I(X_1, X_2; Y|Q) - \delta(\epsilon)
  \]
Gaussian multiple access channel

\[
Y = g_1 X_1 + g_2 X_2 + Z
\]

- \( g_1, g_2 \): channel gains
- \( Z \sim N(0, 1) \)
- Average power constraints: \( \sum_{i=1}^{n} x_{ji}^2(m_j) \leq nP, m_j \in [1 : 2^{nR_j}], j = 1, 2 \)
- Let \( S_j = g_j^2 P, j = 1, 2 \)

Capacity region (Cover 1975, Wyner 1974)

**Theorem 4.4**
The capacity region of the Gaussian MAC is the set of \((R_1, R_2)\) such that

\[
\begin{align*}
R_1 &\leq C(S_1), \\
R_2 &\leq C(S_2), \\
R_1 + R_2 &\leq C(S_1 + S_2)
\end{align*}
\]
Proof sketch

- Let $X_1 \sim N(0, P)$ and $X_2 \sim N(0, P)$ be independent
- Then $\mathcal{R}(X_1, X_2)$ is the set of $(R_1, R_2)$ such that
  \[ R_1 \leq C(S_1), \]
  \[ R_2 \leq C(S_2), \]
  \[ R_1 + R_2 \leq C(S_1 + S_2) \]
- Thus, $\mathcal{C} = \mathcal{R}(X_1, X_2)$ and no time sharing is needed

Proof of achievability:
- Achievability for the DM-MAC with input costs (see NIT Problem 4.8)
- Discretization procedure (NIT 3.4.1)

Proof of the converse: By the maximum differential entropy lemma,
\[ h(Y|X_2, Q) \leq (1/2) \log(2\pi e (S_1 + 1)), \]
\[ h(Y|X_1, Q) \leq (1/2) \log(2\pi e (S_2 + 1)), \]
\[ h(Y|Q) \leq (1/2) \log(2\pi e (S_1 + S_2 + 1)) \]

Comparison to point-to-point coding schemes

- Use point-to-point Gaussian codes
- Treating other codeword as noise:
  \[ R_1 < C(S_1/(S_2 + 1)), \]
  \[ R_2 < C(S_2/(S_1 + 1)) \]
- Time division:
  \[ R_1 < \alpha C(S_1), \]
  \[ R_2 < \bar{\alpha} C(S_2) \]
- Time division with power control:
  \[ R_1 < \alpha C(S_1/\alpha), \]
  \[ R_2 < \bar{\alpha} C(S_2/\alpha) \]
Successive cancellation decoding

\[ y^n \rightarrow M_2\text{-decoder} \quad \hat{m}_2 \quad \text{Encoder} \quad x^n_2(\hat{m}_2) \quad \neg g_2 \quad \text{M}_1\text{-decoder} \quad \hat{m}_1 \]

Extension to more than two senders

**Theorem 4.5**

The capacity region of the \( k \)-sender DM-MAC is the set of \( (R_1, \ldots, R_k) \) such that

\[
\sum_{j \in J} R_j \leq I(X(J); Y|X(J)^c, Q) \quad \text{for every } J \subseteq [1:k] \]

for some pmf \( p(q) \prod_{j=1}^{k} p(x_j|q) \) with \(|Q| \leq k\)

- For \( k = 3 \), the capacity region is the set of \( (R_1, R_2, R_3) \) such that
  \[
  R_1 \leq I(X_1; Y|X_2, X_3, Q),
  R_2 \leq I(X_2; Y|X_1, X_3, Q),
  R_3 \leq I(X_3; Y|X_1, X_2, Q),
  R_1 + R_2 \leq I(X_1, X_2; Y|X_3, Q),
  R_1 + R_3 \leq I(X_1, X_3; Y|X_2, Q),
  R_2 + R_3 \leq I(X_2, X_3; Y|X_1, Q),
  R_1 + R_2 + R_3 \leq I(X_1, X_2, X_3; Y|Q)
  \]

for some \( p(q)p(x_1|q)p(x_2|q)p(x_3|q) \)
Extension to more than two senders

- The capacity region of the $k$-sender G-MAC is the set of $(R_1, \ldots, R_k)$ such that
  \[ \sum_{j \in \mathcal{J}} R_j \leq C \left( \sum_{j \in \mathcal{J}} S_j \right) \quad \text{for every } \mathcal{J} \subseteq [1:k] \]

- For $k = 3$, the capacity region is the set of $(R_1, R_2, R_3)$ such that
  \[
  \begin{align*}
  R_1 &\leq C(S_1), \\
  R_2 &\leq C(S_2), \\
  R_3 &\leq C(S_3), \\
  R_1 + R_2 &\leq C(S_1 + S_2), \\
  R_1 + R_3 &\leq C(S_1 + S_3), \\
  R_2 + R_3 &\leq C(S_2 + S_3), \\
  R_1 + R_2 + R_3 &\leq C(S_1 + S_2 + S_3)
  \end{align*}
  \]

- When $S_1 = S_2 = \cdots = S_k$, the symmetric capacity $C_{\text{sym}} = O(\log(k)/k)$

Summary

- Discrete memoryless multiple access channel (DM-MAC)
- Capacity region
- Time sharing
- Successive cancellation decoding
- Simultaneous decoding is more powerful than successive cancellation
- Time-sharing random variable
- Coded time sharing is more powerful than time sharing
- Packing lemma
- Gaussian multiple access channel
  - Time division with power control achieves the sum-capacity
  - Capacity region achieved via ptp codes, successive cancellation decoding, time sharing
References


